

THE
CIRCLES
OF
PROPORTION
AND
THE HORIZONTAL
INSTRUMENT.

The Former shewing the maner how to
work *Proportions both simple and compound*: and
the ready and easy resolving of *Questions* both in
Arithmetic, Geometrie, & Astronomie: And is newly
increased with an *Additament for Navigation*.

All which Rules may also be wrought with
the penne by *Arithmetic*, and the Ca-
non of Triangles.

The Later teaching how to work most *Que-
stions*, which may be performed by the *Globe*:
and to delineat *Dialls* upon any
kind of *Plaine*.

Invented, and written in Latine by *W. O.*

Translated into English, and set out

for the public benefit,

by WILLIAM FORSTER.

LONDON,

Printed by *Augustine Mathewes*, and are to bee sold
by *Nic: Bourne* at the Royall Exchange. 1633.

SIRGLES

OF

PROPORTION

AND

THE HORIZONTAL

INSTRUMENT

Containing the most
correct and complete
instructions for the
use of the same
in all cases
of surveying
and
navigation



The
author
of
this
work
is
a
member
of
the
Society
of
Surveyors
in
London

Revised, and written in Latin by
Translated into English, and
for the use of
by WILLIAM FORSTER.

LONDON,

Printed by A. MILLAR, in Pall Mall, and are to be sold
by Wm. Forster, in Pall Mall.



TO
THE HONOURABLE
AND RENOWNED FOR
vertue, learning, and true valour,
Sir KENELME DIGBYE
Knight.

SIR,

THE excellent accomplishment: wherewith you are adorned both of vertue, and learning, and particularly in the Mathematicall Sciences, together with the Honourable respect the Author hereof beareth vnto your Worth, and his desire to testifie the same, hath made mee presume to present vnto you, and vnder the happy auspice of your renowned name, to publish to the world this Treatise: the owning whereof though I may not challenge to my selfe, yet the birth and production, whereby it hath a being to the benefit of others, is, as vnto a second parent, due vnto me.

For being in the time of the long vacation
1630, in the Country, at the house of the Reue-

The Epistle Dedicatorie.

rend, and my most worthy friend, and Teacher,
Mr William Oughtred (to whose instruction I
owe both my initiation, and whole progresse in
these Sciences.) I vpon occasion of speech told him
of a Ruler of Numbers, Sines, & Tangents,
which one had bespoken to be made (such as is vsu-
ally called Mr. Gunters Ruler) 6 feet long, to be
vsed with a payre of beame-compasses. " He an-
swered that was a poore invention, and the per-
formance very troublesome: But, said he, see-
ing you are taken with such mechanicall wayes
of Instruments, I will shew you what deuises I
haue had by mee these many yeares. And first,
hee brought to mee two Rulers of that sort, to be
vsed by applying one to the other, without any
compasses: and after that hee shewed mee those
lines cast into a circle or Ring, with another
mouable circle vpon it. I seeing the great ex-
peditenesse of both those wayes, but especially, of
the latter, wherein it farre excelleth any other In-
strument which hath bin knowne; told him, I won-
dered that hee could so many yeares conceale such
vsfull inuentions, not onely from the world, but
from my selfe, to whom in other parts and myste-
ries of Art, he had bin so liberall. " He answered,
" That

The Epistle Dedicatorie.

“ That the true way of Art is not by Instruments,
“ but by Demonstration: and that it is a prepo-
“ sterous course of vulgar Teachers, to begin
“ with Instruments, and not with the Sciences,
“ and so instead of Artists, to make their Schol-
“ lers only doers of tricks, and as it were Jugglers:
“ to the despite of Art, losse of precious time, and
“ betraying of willing and industrious wits, unto
“ ignorance, and idlenesse. That the vse of Instru-
“ ments is indeed excellent, if a man be an Artist:
“ but contemptible, being set and opposed to Art.
“ And lastly, that he meant to commend to me, the
“ skill of Instruments, but first he would haue me
“ well instructed in the Sciences. He also shewed
me many notes, and Rules for the vse of those
circles, and of his Horizontall Instrument,
(which he had proiected about 30 yeares before)
the most part written in Latine. All which I ob-
tained of him leaue to translate into English, and
make publique, for the vse, and benefit of such as
were studious, & louers of these excellent Sciences.

Which thing while I with mature, and diligent
care (as my occasions would giue me leaue) went
about to doe: another to whom the Author in a
louing confidence discovered this intent, vsing
more

The Epistle Dedicatorie

more hast then good speed, went about to proo-
pate; of which vnicely birth, and preuenting (if
not circumuenting) forwardnesse; I say no more:
but aduise the studious Reader, onely so farre to
trust, as he shal be sure doth agree to truthes Art.

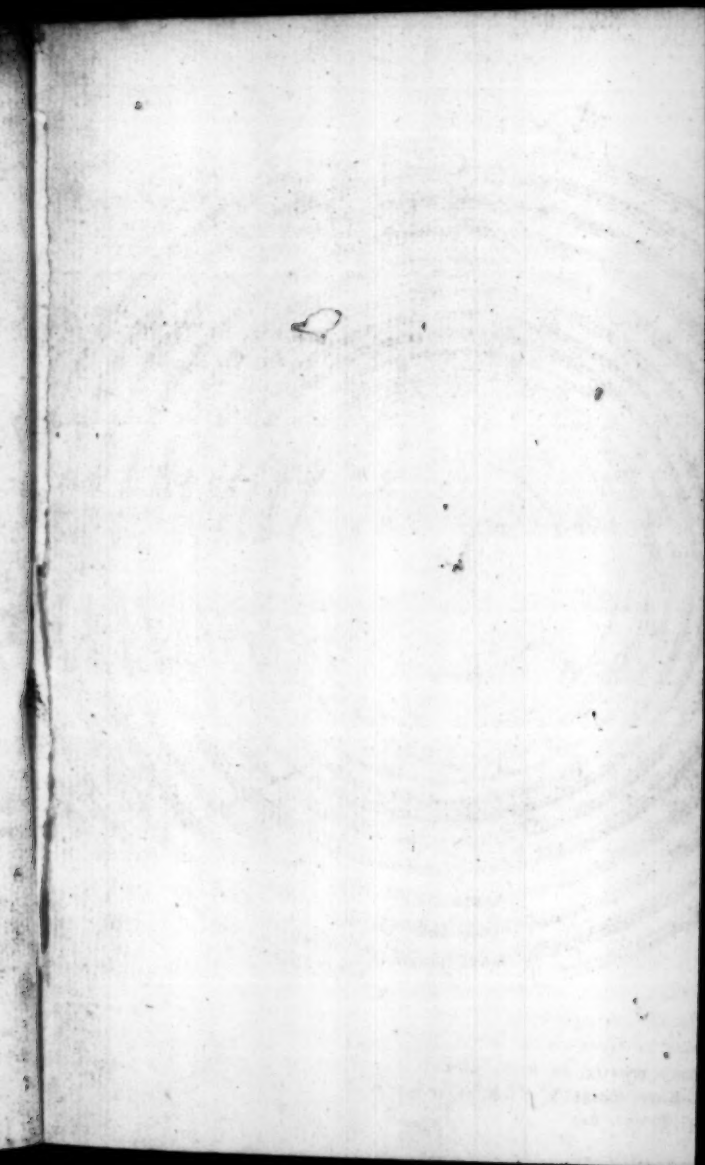
And thus most noble Sir, without any brauing
flourishes, or needlesse multiplying of tautolo-
gized and erroneous precepts, in naked truth, and
in the modest simplicity, of the Author himselfe
(whose knowne skill in the whole Systeme of Ma-
thematicall learning, will easily free him from the
suspicion of hauing the way made for him, and the
subiect vnuailed, to help his sight) I haue not-
withstanding vnder the protection of your courte-
ous fauour, and learned iudgement, persisted in my
long conceined purpose, of presenting this tractate
to the publique view; and light. Wishing withall
vnto you encrease of deserued honor, and happines.
May the 11. 1631.

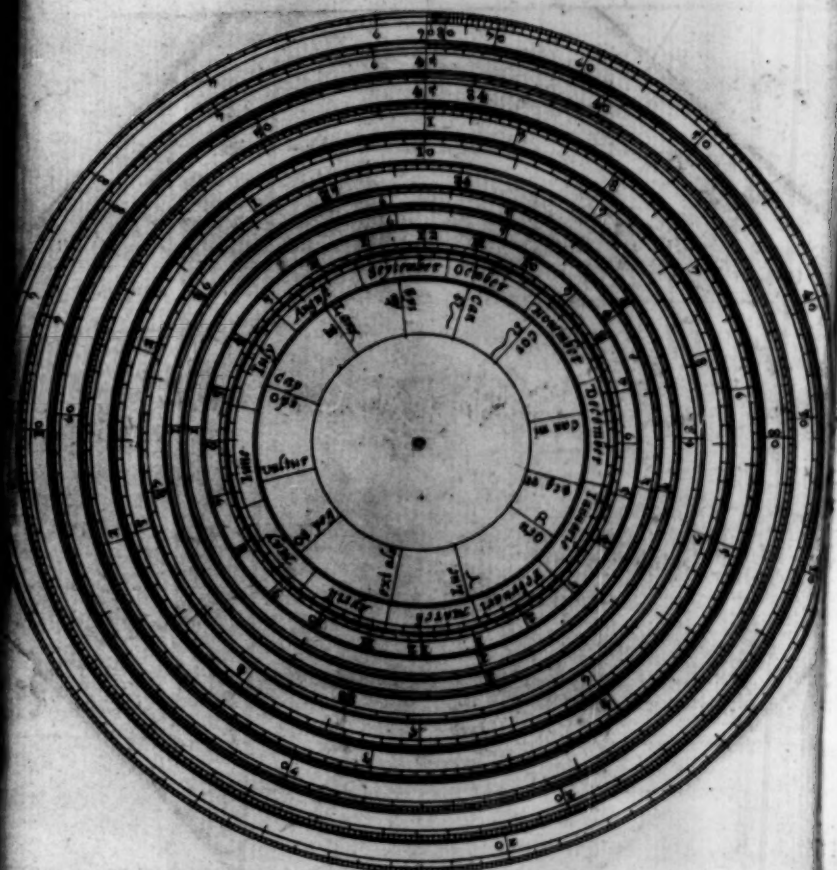
By the honourer

and admirer

of your Worthines,

WILLIAM FORSTER.





Those that desire farther instructions in the use
of these Instruments or other parts of the
Mathematicks may repaire to W Forster
at the Red Bull over against Clements Church
yard with out Temple bar




THE
FIRST PART
OF THIS BOOKE,

Shewing the vse of the *First side*
of the Instrument, for the working of Pro-
portions both simple and compounded, and
for the ready and easie resolving of que-
stions both in *Arithmetique, Geometrie,*
and *Astronomie*, by Calcu-
lation.

CHAP. I.

*Of the Description, and vse of the Circles
in this First side.*

I  Here are two sides of this Instrument. On
the one side, as it were in the *plaine of*
the Horizon, is delineated the *proiection of*
the Sphere. On the other side there are di-
vers kindes of Circles, divided after many severall waies ;

together with an *Index* to be opened after the manner of a paire of *Compasses*. And of this side we will speake in the first place.

2 The *First*, or outermost circle is of *Sines*, from 5 degrees 45 minuts almost, vntill 90. Every degree till 30 is divided into 12 parts, each part being 5 min: from thence vntill 50 deg: into sixe parts which are 10 min: a peece: from thence vntill 75 degrees into two parts which are 30 minutes a peece. After that vnto 85 deg: they are not divided.

3 The *Second circle* is of *Tangents*, from 5 degrees 45 min: almost, untill 45 degrees. Every degree being divided into 12 parts which are 5 min: a peece.

4 The *Third circle* is of *Tangents*, from 45 degrees untill 84 degrees 15 minutes. Each degree being divided into 12 parts, which are 5 min: a peece.

5 The *Sixt circle* is of *Tangents* from 84 degrees till about 89 degrees 35 minutes.

The *Seventh circle* is of *Tangents* from about 35 min: till 6 degrees.

The *Eights circle* is of *Sines*, from about 35 minutes till 6 degrees.

6 The *Fourth circle* is of *Vnauall Numbers*, which are noted with the Figures 2, 3, 4, 5, 6, 7, 8, 9, 1. Whether you vnderstand them to bee single Numbers, or Tens, or Hundreds, or Thousands, &c. And every space of the numbers till 5, is divided into 100 parts, but after 5 till 1, into 50 parts.

The *Fourth circle* also sheweth the *true* or *naturall* *Sines*, and *Tangents*. For if the *Index* bee applied to any Sine or Tangent, it will cut the *true Sine* or *Tangent* in the fourth circle. And wee are to knowe that if the *Sine* or *Tangent* be in the *First*, or *Second circle*, the figures of the *Fourth circle* doe signifie so many thousands. But if the *Sine* or *Tangent* be in the *Seventh* or *Eight circle*, the figures in the *Fourth circle* signifie so many hundreds. And

if

if the *Tangent* bee in the *Sixt circle*, the figures of the *Fourth circle*, signifie so many times tenne thousand, or whole *Radij*.

And by this meanes the Sine of $23^{\circ} 30'$ will bee found 3987 : and the Sine of it's complement 9171. And the Tangent of $23^{\circ} 30'$ will be found 4348 : and the Tangent of it's complement, 22998. And the Radius is 10000, that is the figure 1 with foure cyphers, or circles. And hereby you may finde out both the summe, and also the difference of Sines, and Tangents.

7 The *Fift circle* is of *Equal numbers*, which are noted with the figures 1, 2, 3, 4, 5, 6, 7, 8, 9, 0; and every space is divided into 100 equal parts.

This *Fift circle* is scarce of any use, but onely that by helpe thereof the given distance of numbers may be multiplied, or divided, as neede shall require.

As for example, if the space betweene 11° and $10833+$ bee to bee septupled. Apply the Index vnto $10833+$ in the *Fourth circle*, and it will cut in the *Fift circle* 03476+; which multiplyed by 7 makes 24333 : then againe, apply the Index vnto this number 24333 in the *Fift circle*, and it will cut in the *Fourth circle* $17512+$. And this is the space betweene 11° and $10833+$ septupled, or the Ratio betweene 100, and 108; seven times multiplied into it selfe.

And contrarily, if 17512 bee to bee divided by 7 : Apply the Index vnto 17568 in the *fourth circle*, and it will cut in the *fit circle* 24333 : which divided by 7 giueth 03476+. Then againe vnto this Number in the *Fift circle* apply the Index, and in the *Fourth circle* it will cut vpon $10833+$ for the Septupartion sought for.

The reason of which Operation is, because this *Fift circle* doth shew the *Logarithmes* of Numbers. For if the Index be applyed vnto any number in the *Fourth circle*, it will in the *Fift circle* cut vpon the *Logarithme* of the same number, so that to the *Logarithme* found you prafixe a Character

risticall (as Master Briggs termes it) one lesse then is the number of the places of the integers proposed (which you may rather call the Graduall Number). So the Logarithme of the number 2 will bee found 0.30103. And the Logarithme of the Number 431^6 will bee found 1.63949.

Numbers are multiplied by Addition of their Logarithmes : and they are Divided by Substraction of their Logarithmes.

8 In the middest among the Circles, is a double No-sturnal instrument, to shew the hower of the night.

9 The right line passing through the Center, through 90, and 45 I call the *Line of Unitie*, or of the *Radius*.

10 That *Arme of the Index* which in every Operation is placed at the Antecedent, or first terme, I call the *Antecedens arme* : and that which is placed at the consequent terme, I call the *Consequent Arme*.

CHAP.

CHAP. II.

Of the Operation of the Rule of Proportion: and also of Multiplication, and Division.

1. Theoreme.

IF of three numbers given, the first divide the second and the quotient multiply the third, the product shall be the fourth proportionall, to the three numbers given. $\text{Nam. } a : b :: d : \frac{cd}{a}$

Theoreme. If of three numbers given, the second divide the first, and the quotient divide the third; this later quotient shall be the fourth proportionall, to the three numbers given. $\text{Nam. } \frac{a}{b} : d :: \frac{cd}{b}$

Neither is it materiall whether of the two numbers after the first be second, or third.

2 And note that in *Reciprocal proportion*, that terme by which the question is made; But in *Direct proportion* the terme that is *homogeneous* thereto, is the first terme, or the Antecedent of the first ratio.

3 And therefore out of these foundations thus layd, (if you rightly conceive the nature of the Logarithmes) doth follow the finding out of the fourth proportionall by this Instrument: whereof this is the Rule.

Open the Armes of the Instrument to the distance of the first, and second number: then bring the Antecedent arme, or that which stood upon the first number unto the third, and so the consequent arme, keeping the same opening, will shew the fourth number sought for.

In which operation these foure things are diligently to be considered.

First, in constituting the places of each number in the fourth circle; whether the figures written in the spaces doe signifie Vnites, Tenss, or Hundreds, &c.

Secondly, if that arme which sheweth the fourth proportionall, doe reach beyond the line of the Radius; that then you doe account the fourth in a new circle or degree.

Thirdly, whether the fourth number sought, ought to be greater, or lesser then the third. For if a fourth number bee offered greater then the third, when it should be lesse, or lesse then the third when it should be greater; it is a signe that that number doth appertaine to a circle of another degree.

Fourthly, that looke what true distance was betweene the first and second, that the same bee supposed betweene the third and the fourth, and also on the same part.

4 And for because *Multiplication* and *Division*, have a certaine implicite proportion: we will speake of them in the first place.

5 In *Multiplication*, As an *Vnite* is to one of the factores (or numbers to be multiplied:) so is the other of the factores, to the product. $1 \cdot 5 :: 4 \cdot 20$.

And the product of two numbers shall have so many places as there be in both the factores, if the lesser of them exceede

exceede so many of the first figures of the product: But if it doe not exceede, it will have one lesse.

6 And in *Division*, As the *Divisor* is to an *Unit*; so is the *Dividend*, to the *Quotient*. $5 : 1 :: 4 : \frac{4}{5}$.

And the *Quotient* shall have so many places, as the *Dividend* hath more then the *Divisor*, if the *Divisor* exceede so many of the first figures of the *Dividend*: but if it doe not exceede, it shall have one place more.

7 Wherefore let this rule bee still carefully kept in minde: that In *Multiplication* the first terms of the *implied proportion* is evermore 1: And in *Division*, the first terms is the *Divisor*.

And thus much concerning the operation of *Proportion*, *Multiplication*, and *Division*, I thought meete to admonish, least hereafter in *Multiplying*, or *Dividing*, or seeking out a fourth proportional, wee be constrained to repeat the same things many times over.

8 An example of *Multiplication*. How many pence are there in 47^{li}. 9^{sh}? For because 1 shilling contains 12 pence, and 1 pound contains 20 shillings, that is 240 pence: you shall multiply 47 by 240, and 9 by 12, and then adde together the products.

In the first *Multiplication*.

$$1 \cdot 47 : : 240 \cdot 11280 \cdot$$

For set the *Armes* of the *Index* at 1 and 47 in the fourth circle; and then bring the *Antecedent arme* (which stood at 1) unto 240, and the *Consequent arme* will shew 11280.

Again in the second *Multiplication*.

$$1 \cdot 9 : : 12 \cdot 108 \cdot$$

For set the two *Armes* of the *Index* at 1 and 9 in the fourth circle; and bring the *Antecedent arme* unto 12, and the *Consequent arme* will shew 108. Lastly, adde together 11280 and 108 and the summe 11388 will be the number of pence contained in the said summe of 47^{li}. 9^{sh}.

9. An

9 An example of *Division*. How many pounds, and shillings are in 11388 pence? Divide 11388 by 240: the division is thus.

$$240 \cdot 1 :: 11388 : 47\frac{1}{2} -$$

For set the two Armes of the Index at 240, and 1 in the fourth circle: and then bring the Antecedent arme (which stood at 240) unto 11388; and the Consequent arme will shew 47 and almost an halfe.

But how many shillings that excesse doth containe will appeare, if first you finde by Multiplication that 11380 pence are contained in 47^h: which subducted from 11388 there will remaine for the excesse 108 pence. Afterwards by division you may seeke how many shillings are in 108 pence: the diuision is thus.

$$12 \cdot 1 :: 108 \cdot 9 \cdot$$

For set the Armes of the Index at 12 and 1: then bring the Antecedent arme (which stood at 12) unto 108; and the Consequent arme will shew 9.

9 Any Fraction given may bee reduced into *Decimal parts*, thus.

Set the Antecedent arme of the Index at the Denominator of the Fraction given, in the fourth circle, and the Consequent arme at the Numerator, then keeping the same distance, bring the Antecedent arme unto 1, and the consequent arme will shew the *decimal parts*.

$$\text{So } \frac{1}{12} \text{ is } 0\frac{1}{2}5. \text{ And } \frac{2}{11} \text{ is } 0\frac{1}{3}05 -$$

CHAP. III.

*Now follow certaine examples of
Proportion.*

Example I.

F 54 Elnes of Hol and bee sold for 96 shillings, for how many shillings was 9 elnes sold? The termes given are

$$54 \cdot 96 :: 9 \cdot$$

According to the 2 Chap. 3 Sect. Set one of the armes of the Index at the Antecedent terme 54 in the fourth circle, and the other arme at the consequent terme 96: then keeping that distance, bring the Antecedent arme unto 9; and the consequent arme beyond the *line of the Radius* will shew 16 for the fourth proportionall, according to the *considerations* in the 2 Chap. Sect. 3.

Therefore

$$54 \cdot 96 :: 9 \cdot 16 \cdot$$

are proportionals. And 16 shillings is the price of 9 Elnes.

Example II. If 108 bushels of corne be sufficient for a company of Souldiers keeping a Fort, for 36 dayes, How many dayes will 12 bushels suffice that same number of Souldiers? The termes given are

$$108 \cdot 36 :: 12 \cdot$$

Set one Arme of the Index at the Antecedent terme 108 in the fourth circle, and the other Arme at the consequent terme 12, (being mindfull of the *considerations* in the 2 Chap. 3 Sect.) then keeping that same distance,

C

bring

bring the Antecedent arme vnto 36 ; and the consequent arme will shew 4.

Therefore

$$108 . 36 :: 12 . 4 .$$

shall bee proportionals. And 4 is the number of dayes fought for.

Example III. There is layd vp in a Fort so much corne as will suffice for 756 Souldiers which keepe that Fort, for 196 dayes : how many Souldiers will that same corne suffice for 364 dayes ?

The Proportion is reciprocall , therefore the termes given are

$$364 : 756 :: 196 .$$

Set one Arme of the Index at the Antecedent terme 364 , and the other Arme at the consequent 196 : and keeping the same distance , bring the Antecedent arme vnto 756 ; and the consequent arme will shew 407 4 : And therefore for so many Souldiers will the corne laid vp suffice for 364 dayes, or 13 monthes.

Example IIII. There is a Tower whose height I would measure with a Qadrant.

I take two Stations in the same right line from the Tower : and at either Station having observed the height through the sights, I finde that the perpendicular cutteth in the nearer Station 28 degrees 7 minutes almost : and in the further Station 21 degr. 53 min. almost : and betwene both the Stations, the distance was 76 feet.

The Rule of measuring heights by two Stations is contained in these Theoremes.

Thur. As the difference of the Tangents of the arches cut in either station, is to the distance betwene

tweenc the stations; so is the Tangent of the lesser arch, to the nearer distance from the Tower. Again

Theor. As the Radius is to the Tangent of the greater arch; so is the nearer distance found, to the height.

And therefore because by the 1 Chap. 6 Sect. the Tangents of the arches $28^{\circ} 7'$ —, and $21^{\circ} 58'$ —are 5342, and 4032 whose difference is 1310; the proportions will be

First, 1310 . 76 :: tang. $21^{\circ} 58'$ — . 234 .

Wherefore 234 feet is the nearer distance.

Second Radius . tang. $28^{\circ} 7'$ — :: 234 . 125 .

Wherefore 125 feet is the height sought for.

Example V. To finde the *Declination of the Sunne* the 9th day of May.

The place of the Sunne for every day, may be had nere enough out of this Table, by Adding vnto the place of the Sun in the beginning of that moneth so many degrees, as there are dayes past in that moneth: But if the number of degrees exceed 30, the excess is to be accounted in the Signe next following.

Wherefore the 9th of May the place of the Sun is $\varnothing 20 + 9$, that is $\varnothing 29$: which is 59 degr. distant from the next Equinoctial point.

The place of the
Sunne, in the begin-
ning of every Month.

January	Υ	21
February	\varnothing	22
March	\mathbb{X}	20

April	Υ	21
May	\varnothing	20
June	\mathbb{H}	19

July	\mathbb{S}	18
August	Ω	18
Septemb.	\mathbb{R}	18

October	\mathbb{A}	17
Novemb	\mathfrak{m}	18
Decemb	\mathbb{Z}	19

These things being knowne, the Rule is delivered in this Theoreme.

Theor. As the Radius is to the sine of the sunnes distance from the next Equinoctiall point; so is the sine of the sunnes greatest declination, to the sine of the declination sought for.

The proportion will be

$$\text{Radius} \cdot \text{sine } 59^\circ :: \text{sine } 23^\circ, 30' \cdot \text{sine } 19^\circ, 59'.$$

And so much is the Declination sought for.

Example VI. To find the Right ascension of the Sunne, the 9th day of May.

Suche the place of the Sunne for the day proposed in the former Table; and the Sunnes distance from the next Equinoctiall point, as in the former example.

These things being knowne, the Rule is by one of these two Theoremes.

Theor. As the Radius, is to the sine of the complement of the sunnes greatest declination; so is the tangent of the sunnes distance from the next Equinoctiall point, to the tangent of the distance of the right ascension of the sunne, from the same Equinoctiall point. Or

Theor. As the tangent of the greatest declination of the Sunne, is to the Radius; so is the tangent of the declination of the sunne for the time proposed, unto the sine of the right ascension of the sunne from the next Equinoctiall point.

The proportions will be either

$$\text{Radius} \cdot \text{sine } 66^\circ, 30' :: \text{tang. } 59^\circ \cdot \text{tang. } 56^\circ, 46'.$$

$$\text{Or tang. } 23^\circ, 30' \cdot \text{Radius} :: \text{tang. } 19^\circ, 59' \cdot \text{sine } 56^\circ, 46'.$$

Quæ.

CHAP. III.

Of Continuall proportion, Or of Progression Geometrical.

THE Ratio of a Progression is the quotient of the consequent terme divided by his antecedent. And therefore in the Instrument it is the distance taken between the termes in the fourth circle, by the opening of the Index.

To Double, Triple, or Multiply how often soever any Ratio given, is nothing else but so often to put together the said space or distance between the termes, as is shewed in Chap 1 Sect 7.

As for example, if the Ratio 60, to 65 be proposed to be *sempled*.

Set the Armes of the Index at 60, and 65: and then with the same opening bring the Antecedent arme which was at 60, unto 1, and the consequent arme will cut 1.0408 in the fourth circle, and 03.476 in the fifth circle: this latter number being multiplied by 7 maketh 245.33: unto which number in the fifth circle applying the Index, it will in the fourth circle cut 1720, which is the multiplied number sought for.

But because in a little Instrument, the arch cut in the fifth circle, cannot be estimated exactly: and a small error in the beginning often repeated, by multiplying is made great: it is the most safe way, to take the *Logarithmes* of the termes of the Ratio out of the Canon, and to multiply them by the number given: As I have done in these examples.

Logarithmes taken out of the Canon.

100	2 . 0000000
104	2 . 0170333
105	2 . 0211893
106	2 . 0253059
106½	2 . 0280287
107	2 . 0291838
107½	2 . 0314085—
108	2 . 0334238—
108½	2 . 0347621
108¾	2 . 0354297+
108½	2 . 0364293—
109	2 . 0374265
110	2 . 0383937

4

Or if the *Canon* be wanting ; you may come neerer the marke, if that first single opening of the Index being kept, and the Antecedent arme set at 1 ; you transerre the Antecedent Arme, unto that place which the consequent arme doth cutt ; and the consequent arme will cut the same space *duplicated*. Then holding the consequent arme in that same place, open the Antecedent arme unto 1 . and afterward with that *duplicated opening*, bring the Antecedent arme to the *duplicated space*, and the consequent arme will cut the *space quadrupled*. Thirdly, bring the Antecedent arme unto the *quadrupled space* and the consequent arme, keeping that *duplicated opening*, will cut the *space sexupled*. Lastly, having againe taken a single opening, bring the Antecedent arme unto the number, or *space sexupled* ; and the consequent arme will shew the *Ratio* sought for *septupled*.

And this manner of working may bee observed for as many Multiplications as you please of any *Ratio* given.

3. The Ratio, and first term^{of a Progression} being given,
to continue the same unto any number
of termes.

Open the Armes of the Index, the one unto the Antecedent of the ratio given, and the other unto the consequent: then the same opening being kept, bring the Antecedent Arme vnto the first terme given, and the consequent arme will shewe the second terme: againe bring the Antecedent arme vnto the second terme found, and the consequent arme will shew the third. After that bring the Antecedent arme unto the third terme found, and the consequent arme will shew the fourth. And in this manner you may proceed as farre as you please.

As for example, If a Progression in the ratio 2 unto 5, beginning at 8, Or if a Progression in the ratio 100 unto 108, beginning at 5, is to be instituted; the termes in either progression will be as followeth.

1 2 5 :: 8 . 20

2 20

3 50

4 125

5 3125

6 78125

7 1953125

1 100 108 :: 5 . 54

2 54

3 5184

4 6129856

5 68114448

6 7146640384

7 723427161472

4 *Theor*: The *ratio* of any former terme, in a row of continuall proportionals, vnto any of the termes following, is equall to the *ratio* of the first terme vnto the second, multiplied into it selfe according to the distance of that latter terme from the former.

As for example, The *Ratio* 54 vnto 6²⁰²⁴⁴¹ which is the third terme from it, is equall to the *ratio* of 100 vn. to 108 triplicated, Or as the Cube of 100 vnto the Cube of 108. Wherefore

5 The *Ratio*, and the first terme of the *Progression* being given, to finde out any other terme required.

First, multiply the *ratio* given into it selfe, according to the distance of the terme sought from the first terme, by the 2 sect: then say

As 1 is to that multiple found; so is the first terme, to the terme sought for.

Example, What will be the amount of 26th, in 7 yeares by *Interest upon Interest*, after the rate of 20 pence in the pound?

Because 1 pound which is 20 shillings containeth 60 groates, and 20 pence containe 5 groates, the rate of the *Interest* will bee 60 vnto 65, Or 100 vnto 108¹¹ = 1 But the first terme is given 26, vnto which there are to be acquired 7 other termes in continuall proportion.

First, by the 4 Chap: and 2 Sect. Let the *ratio* given be septuplicated, that is multiplied seuenfold into it selfe, which will be 1⁷⁷¹². Then let the consequent arme of the Index vnto the septuplicate number 1⁷⁷¹², and open the Antecedent arme vnto 1; and keeping the same opening

ning, bring the Antecedent arme unto 26; and the conſequent arme will ſhew 45 $\frac{17512}{1}$ the amount fought for.

6 The Ratio, and any other terme of the Progreſſion being given, to finde the firſt terme.

Fiſt multiply the Ratio given into it ſelfe according to the difference of the terme given from the firſt terme. Then ſay

As that multiple is unto 1; ſo is the terme given, unto the firſt terme.

Example, what ſumme in 7 yeares did amount unto 45 $\frac{17512}{1}$ by Intereſt upon Intereſt after the rate of 100 unto 108 $\frac{33}{100}$ + ?

Fiſt the ratio being ſeptuplicat, by the 4 Chap. 2 Sect. will be $\frac{17512}{1}$. Then ſetting the Antecedent arme of the Index to that ſeptuple $\frac{17512}{1}$, open the conſequent arme unto 1: and keeping the ſame opening, bring the arme unto 45 $\frac{17512}{1}$: and the conſequent arme will ſhew 26, which was the ſtocke, or ſumme of money, from which that amount did ariſe.

7 The Ratio, the Firſt terme, and any other terme of a Progreſſion being given, to finde how many places the terme given is from the firſt terme.

Fiſt ſay, *As the firſt terme is unto 1; ſo is the other terme given, unto the ratio multiplied into it ſelfe according to the diſtance of that terme from the firſt.*

Wherefore according to 1 Chap. 7 Sect. and 3 Chap. 2 Sect. by helpe of the fiſt circle, ſee how often the ratio given, is contained in that multiple found.

Example, In how many yeares did 26^{li}, by Intereſt

upon Interest after the rate of 100 unto 108¹³ + increase unto 45^{53 12 li}?

First let the Antecedent arme of the Index at 26, and the consequent arme at 1: and keeping the same opening bring the Antecedent arme unto 45^{53 12}, and the consequent arme will shew 17¹²: to which in the fifth circle answereth 24333. Then because unto the consequent terme of the ratio 108¹³ + there agrees in the fifth circle 03476, divide 24333 by 03476, and the quotient will be 7, the number of yeares sought for.

8 The First terme, and any other terme of the Progreſſion being given to finde the ratio of the Progreſſion.

First ſay, As the first terme is unto 1: ſo is the other terme given, unto the ratio multiplied into it ſelfe according to the diſtance of that terme from the firſt.

Wherefore according to Chap. 1. Sect. 7, by helpe of the fifth circle, Let the multiple found be divided by the diſtance of the terme from the firſt.

Example, 26^{li} by Interest upon Interest in 7 yeares amounted unto 45^{53 12}: what was the ratio of the Interest compared unto 100?

First ſet the Antecedent arme of the Index at 26, and the consequent arme at 1: and then keeping the ſame opening, bring the Antecedent arme unto 45^{53 12}: and the consequent arme will ſhew 17¹²: unto which in the fifth circle answereth 24333: and this number being divided by 7 will give 03476 +: unto which agreeth in the fourth circle 108¹³ + the consequent terme of the Interest ſought for.

9 Two numbers being given to finde as many Middle proportionals betweene them as you will.

Divide

Divide the distance of the greater number given from the lesser in the fourth circle justly taken, according to Chap. 1. Sect. 7, by helpe of the fift circle into equall segments, one more then are the number of Midle proportionals sought for. All these segments added orderly to the first terme, doe distinguish the termes of the Progression which you seeke.

Example, Let there be foure *Midle proportionals*, sought out betweene 8 and 19⁹⁰⁶⁵⁶.

Apply the Index unto 8 in the fourth circle, and it will cut in the fift circle 9039: also set the Index unto 19⁹¹ in the fourth circle, and it will cut in the fift circle 2989; which number because it reaches beyond the Unite line, is indeede 12989, according to Chap. 1. Sect. 7, and so is the distance justly taken. Then subducting 9039 from 12989, there will remaine 3950: which divided by 4 + 1, the quotient will give 790. wherefore 9039 + 790, scil. 9829 in the fift circle doth agree with 9¹⁶ in the fourth circle, which is the *first middle proportionall*. And 9039 + twise 790, scil. 10619 in the fift circle doth agree with 11⁵² in the fourth circle, which is the *second middle proportionall*. And in this manner 13⁸²⁴ and 16⁵⁸⁸⁸ will be found the *third* and *fourth middle proportionals*.

10 *Theor.* If from the Ratio given, being multiplied in it selfe according to the number of termes, you subduct 1, and multiply the remainder, by the antecedent of the ratio. It will be

As the difference of the termes of the ratio, is unto the product; so is the first terme, to the sum of the Progression.

As for example if the *Ratio* of the Progression be R to S: and the *difference* of R taken out of S be D. and the *first terme* of the Progression a: and the whole *summe* of the termes Z it shall be,

D a

D . rat.

$$D \cdot \text{rat. mul}^{\text{ta}} - 1 \text{ in } R :: a \cdot Z.$$

Which is the very Theoreme it selfe expressed in *Symboles of words*: that it may more easily be fixed in the phantasie. Which proportion also wee must consider, doth hold both *alternely*, and *conuersely*.

This Theoreme may otherwayes be expressed, by the equality which the product of the two middle termes hath to the product of the two extreames, thus

$$\text{Ratio multiplicata} - 1 \text{ in } R \text{ in } a = Z \text{ in } D.$$

This manner of setting downe Theoremes, whether they be Proportions, or Equations, by Symboles or notes of words, is most excellent, artificiall, and doctrinall. Wherefore I earnestly exhort every one, that desireth though but to looke into these noble Sciences Mathematicall, to accustome themselves unto it: and indeede it is easie, being most agreeable to reason, yea even to sence. And out of this working may many singular consecutaries be drawne: which without this would, it may be, for ever lye hid in this present proportion: because it is,

$$D \cdot \text{rat. mul}^{\text{ta}} - 1 \text{ in } R :: a \cdot Z \text{ . wherefore}$$

$$\text{Rat. mul}^{\text{ta}} - 1 \text{ in } R \cdot D :: Z \cdot a \text{ . And}$$

$$a \cdot Z :: D \cdot \text{rat. mul}^{\text{ta}} - 1 \text{ in } R$$

These are exceeding easie: but this following is more difficult, and requireth attention.

In the former *aquation* it was

$$\text{Rat. mul}^{\text{ta}} - 1 \text{ in } R \text{ in } a = Z \text{ in } D$$

Now because $\text{Rat. mul}^{\text{ta}} - 1 \text{ in } R \text{ in } a,$

$$\text{and } \text{Rat. mul}^{\text{ta}} \text{ in } R \text{ in } a = R \text{ in } a,$$

$$\text{and } \text{Rat. mul}^{\text{ta}} \text{ in } R = R \text{ in } a,$$

and

and Rat. mult^a in $\alpha - \alpha$ in R,
are equal one to another, and also to Z in D, these equations shall also be conf. tarius.

$$\frac{\text{Rat. mult}^a - 1 \text{ in R in } \alpha}{D} = Z.$$

$$\text{and } \frac{Z \text{ in D}}{\alpha} = \text{rat. mult}^a \text{ in R} - R.$$

$$\text{and } \frac{Z \text{ in D}}{R} = \text{rat. mult}^a \text{ in } \alpha - \alpha.$$

$$\text{and } \frac{Z \text{ in D} + R \text{ in } \alpha}{R \text{ in } \alpha} = \text{rat. mult}^a.$$

And besides these many more. The practise whereof I leave to the industry of the studious Reader, especially having delivered the whole Art of such operations in my *Clavis Mathematica*.

Some of these I have occasion to use in the sections following.

11. *Therefore the Ratio of a Progression, and the first terme, and the number of termes being given, to finde the summe of the whole progression.*

For this operation the rule, or Theoreme last before serveth: for by it

$$\frac{\text{Rat. mult}^a - 1 \text{ in R in } \alpha}{D} = Z.$$

D 3

Example.

Example. If an *Annuity* of 5^{li}, be detained 7 yeares, what will be the *amount* thereof by interest upon interest after the *rate* of 100 unto 108?

Now because the *Amount* is the *summe of the Progression*, whereof the first *terme* is the *annuity*, Multiply the ratio into it selfe according to the number of yeares, by Chap. 4. Sect. 2, and it will be 1713^{82} : from which if you subduct an vnite, there remaines 0711^{82} , which multiplied by 100 maketh 711^{82} : then let the Armes of the Index at 1 and 711^{82} ; and bring the Antecedent arme which stood at 1, unto 5 the first terme: and the consequent arme will cut 356^{215} (that is Rat. $\text{mult}^a - 1$ in R in α) and lastly this number being divided by 108—100, scil. by 8, the quotient will give 44^{6140} , for the *summe of the whole progression*: and so much is the *amount* sought for.

11. The Ratio, the Number of Termes, and the Summe of a Progression being given, to find the first terme.

By the *converso* of the foregoing Theoreme, it is manifest, that

$$\text{Rat} : \text{mult}^a - 1 \text{ in R} \cdot D : : Z \cdot \alpha \cdot$$

The declaration in words was in the 10 Sect.

Example. If an *Annuity* detained 7 yeares by Interest upon Interest, after the rate of 100 unto 108, did increase unto 44^{6140} . how much was that *Annuity*?

Multiply the *Ratio* into it selfe according to the number of yeares (*per cap. 4. sect. 2.*) and the product will be 1711^{82} : from which if you subduct an Vnite, there remaines 0711^{82} : this being multiplied by 100 doth make 711^{82} . Then say

$$711^{82} \cdot 8 : : 44^{6140} \cdot 5 \cdot$$

for

for the First terme : which was the *Annuitie* sought for.

- 13 The *Ratio*, the *First terme*, and the *Summe of the Progression* being given, to find the *Number of termes*.

By the Theoreme in the 10. Sect. it was

$$a . Z : : D . \text{rat. mul}^{\text{ta}} - 1 \text{ in R.}$$

Wherefore set the Antecedent arme of the Index unto the Antecedent terme of the ratio, and the consequent arme unto the Summe of the Progression: and with that same opening, bring the Antecedent arme unto the difference of the termes of the ratio; and the consequent arme will shew a number (that is Rat. mul. — 1 in R;) which if you divide by the Antecedent terme of the ratio, and unto the quotient adde an Vnite, you shall have the ratio multiplied into it selfe according to the number of termes. Therefore taking the distance betweene the termes of the ratio, with the armes of the Index, measure by helpe of the fifth circle (*per Cap. 4. Sect. 7.*) how often that distance may be found in the multiplied ratio: for so many are the termes of the progression.

Example. If an *Annuitie* of 5^{li} detained by Interest upon interest after the rate of 100 to 108, increased unto 44⁶¹⁴⁰^{li}. How many yeares was the *Annuitie* detained?

Set the Antecedent arme of the Index at 5, and the consequent arme at 44⁶¹⁴⁰ and with the same opening bring the Antecedent arme unto 8, and the consequent arme will shew 71²⁸² (that is Rat. mul^{ta} — 1 in R:) this number being divided by 100, will be 0,71²⁸²: and if unto the quotient you adde 1, you shall have 1,71²⁸² (the ratio multiplied into it selfe according to the number of yeares:) unto which in the fifth circle answereth 2338:
but.

but unto 108 in the fifth circle there answereth 0334. divide therefore 2338 by 0334; and the quotient will be 7, for the number of yeares sought for.

Or such questions may be more easily performed by this Theoreme, which the industrious Reader may by himselfe practise.

$$\frac{Z \text{ in } D \div R \text{ in } a}{R \text{ in } a} = \text{rat. multa}$$

14 Theor. If the summe of the whole progression be divided by the ratio multiplied into it selfe according to the number of termes, the quotient will be the first terme; and that summe given will be the last terme, of another progression, having the same ratio but one terme more.

15 And because a summe of money the amount whereof in any number of yeares given, by Interest upon interest, doth equall an Annuity so long detained, is equivalent to the same Annuity; and the amount of an Annuity is the summe of a Progression continued from that Annuity. If therefore an Annuity for any number of yeares be divided by the ratio multiplied into it selfe according to the number of yeares; the quotient will be the just price of an Annuity to endure for so long. And because by the 10 and 11 Sect. it hath beene shewed that

$$\frac{\text{Rat. multa} - 1 \text{ in } R \text{ in } a}{D} = \text{to the amount.}$$

Therefore by the 14 Sect. $\frac{\text{Rat. multa} - 1 \text{ in } R \text{ in } a}{\text{Rat. multa in } D}$

will be equall to the price of an Annuity in ready money, which shall be the Rule for the operation following.

Where-

Wherefore also

Rat. mul^{ta} — 1 in R in α = Rat. mul^{ta} in D in Pret.

which proportion is thus enuntiated in words.

Theor. If from the Ratio multiplied into it selfe according to the Number of yeares you subduct an Vnite, and the remainder be multiplied continually by the Antecedent of the Ratio, and the Annuitie it selfe: *And againe,* If the Ratio multiplied into it selfe, according to the number of yeares be multiplied continually by the difference of the termes of the Ratio, and by the Price: both those products will be equall.

Example. An Annuitie of 5^{li}, to endure for 7 yeares, is to be sold: what is it worth in ready money, after the Rate of 100 unto 108?

The Ratio multiplied into it selfe according to the number of yeares (*per Cap. 4 Sect. 2*) is $1,7138$: subduct 1, and there will remaine $0,7138$: which multiplied by 100 maketh $71,38$. Then set the Antecedent arme of the Index at 1, and the consequent arme $71,38$: and keeping that same opening, bring the Antecedent arme unto 5; and the consequent arme will shew $356,915$; keepe this (for it is $\frac{\text{rat. mul.} - 1 \text{ in R in } \alpha}{D}$). After that let the Ante-

cedent arme of the Index at 1, and the consequent arme unto the multiple ratio $1,7138$; and with the same opening bring the Antecedent arme unto 8; and the consequent arme will cut $13,7106$, keepe this number also (for it is Rat. mul. in 1). Lastly, place the Antecedent arme of the Index at $13,7106$, and the consequent arme at 1; and with the same opening bring the Antecedent arme vnto $356,915$: and the consequent arme will shew $26,2321$.

E

which

which is the iust Price of an Annuitie of 5^{li} in readie money.

16 And by the last præcedent Theoreme or Rule, also

$$\frac{\text{Rat. mul. in D in Pret} = a}{\text{Rat. mul.} - 1 \text{ in R}}$$

which Theoreme may bee enuntiated in words as was there shewed.

Example. An Annuitie for 7 yeares is bought for 26⁰²li. after the rate of 100 unto 108, by Interest upon interest: how much was that Annuitie?

The Ratio multiplied into it selfe for the number of 7 yeares (*per Cap: 4, Sect. 2*) is 1.7118: which multiplied continually by 8, and by the price, doth make 356⁰²5. Divide this number found, by 7118², (which is the *multiplier ratio*) it selfe lesse by an Vnite, and multiplied by 100: and the quotient will be 5^{li}, the Annuitie sought for.

17 Also by *Rationation* from that præcedent rule will follow this proposition

$$\frac{R \text{ in } a}{R \text{ in } a - \text{Pret. in D}} = \text{Rat. mul.}^a$$

which is thus enuntiated in words.

Theor. If the product of the Antecedent of the ratio multiplied by the Annuitie be divided by it selfe, being diminished by the product of the difference of the termes of the ratio multiplied by the Price: the quotient will bee equall to the ratio multiplied into it selfe according to the number of yeares. As

If the ratio be 100 unto 108; the Annuitie 5^{li}: and the price thereof 26⁰²li. Set the Antecedent arme of the

the Index at 1, and the consequent arme at 8, the difference of the termes of the ratio: and with the same opening bring the Antecedent arme unto 261^{212} —: and the consequent arme will shew 208^{256} : which subducted from 500, there will remaine 291^{744} , for the *divisor*. Set therefore the Antecedent arme of the Index at the *divisor* 291^{744} , and the consequent arme at 1: and with the same opening, bring the Antecedent arme unto the *dividend* 500: and the consequent arme will shew 1^{718} , which is the *ratio* multiplied into it selfe according to the number of yeares. And by this number so found it will be easie (by helpe of the fifth circle) the *ratio* of the Interest being given, to finde the continuance of the Annuities.

Example. An Annuity of 5^{li}, was bought for 260^{011} —, after the rate of 100 unto 108: how many yeares is it to last?

First seeke out the ratio multiplied into it selfe according to the number of yeares, which will be 1^{718} , according as was even now shewed in this Sect. to this in the fifth circle there answereth 2338: but unto 108 there answereth in the fifth circle 0334. Divide therefore 2338 by 0334; and the quotient will be 7 for the number of yeares sought for.

CHAP. V.

*Of the Quadrating, and Cubing of Numbers,
Sides or Rootes: and of the Extraction of
the Quadrate, and Cubic side, or roote, out
of Numbers, or Powers given.*

I *F a number, side, or roote be multiplied into
it selfe, the product will be a Quadrat. And
if a quadrate bee multiplied into his owne
side, or roote, the product will be a Cube.*

Wherefore $1 \cdot N :: N \cdot Q$.

and $1 \cdot N :: Q \cdot C$.

2 If therefore a number be given to be Quadrated.

Set the Antecedent arme of the Index at 1, and the consequent arme at the number given: then With the same opening bring the Antecedent arme to the number given; and the consequent arme will shew the Quadrates thereof.

And the number of figures in a Quadrat of a single root (or which doth not exceede 9) is easily found out by those Rules, that have beene before delivered concerning Multiplication. But if a side or root consist of more figures then one; for each figure after the first it acquireth two more places of figures. And if any of the figures of the root given be decimal parts, cut off from the Quadrat found, twice so many of the last figures for decimals.

Example 1. The Quadrat of the side 7 is required. Say

$$1 \cdot 7 :: 7 \cdot 49$$

Set therefore the Antecedent arme of the Index at 1, and the consequent arme at 7; and with that opening bring

bring the Antecedent arme unto 7; and the consequent arme will shew 49 which is the Quadrat sought for.

Example II. The Quadrat of the side 57 is required.

Set the Antecedent arme of the Index at 1; and the consequent arme at 57: then with that same opening bring the antecedent arme unto 57; and the consequent arme will shew 3249, which is the Quadrat sought for, consisting of foure places.

Example III. The Quadrat of the side, or root 570 is required.

Having found as before, 3249 for the quadrat of the side 57: put thereunto two circles: and it will bee 324900, the quadrat sought for.

Example IIII. The quadrat of the side 574 is required.

Set the Antecedent arme of the Index at 1, and the consequent arme at 574; and with the same opening bring the Antecedent arme unto 574; and the consequent arme will shew 329476, the quadrat sought for consisting of sixe figures: but the two last figures cannot at all be discerned by the Instrument.

3 If a number be given to be Cubed.

Set the Antecedent arme of the Index at 1, and the consequent arme at the number given; and with that same opening, bring the Antecedent arme unto the number given; and the consequent arme will shew the Quadrat; then bring the Antecedent arme unto the Quadrat, and the consequent arme with that same opening will shew the Cube of that side given.

The number ^{of} figures in a Cube of a single side, or root which doth not exceede 9, is easily found by that which hath beene before delivered concerning Multiplication:

But if the side, or root consist of more figures then one; for each figure after the first it obtaineth three more places of figures. And if any of the figures of the root given be *decimal parts*, cut off from the *Cube* found thrice so many of the last figures for *decimal parts*.

Example. The Cube of the side, or root 7 is required.

Say, $1 \cdot 7 :: 7 \cdot 49 \cdot$

again $1 \cdot 7 :: 49 \cdot 343 \cdot$

Set therefore the Antecedent arme of the Index at 1, and the consequent arme at 7; and with that opening bring the Antecedent arme unto 7, and the consequent arme will shew 49 the quadrat thereof: Then set the Antecedent arme at 49; and the consequent arme (with that first opening) will shew 343, which is the desired Cube of the side proposed.

Another example, The Cube of the side, or root 57 is required.

Set the Antecedent arme of the Index at 1, and the consequent arme at 57; and with that same opening bring the Antecedent arme unto 57; and the consequent arme will shew the quadrat 3249. Then set the Antecedent arme at 3249; and the consequent arme keeping the former opening will shew 185193, which is the required Cube of that side proposed, consisting of six places: but the two last figures cannot be known by the Instrument.

Example III. The Cube of the side 570 is required.

Having found as before the Cube of the side, or root 57 to be 185193: put thereunto three circles; and it will be 185193000 the Cube sought for.

Examples of greater Cubes, it will be needlesse to set downe.

4 The Extraction of the Square, or Quadrat root, or side, is done by helpe of the fifth circle, after this manner.

Set the Index at the Quadrat proposed; and of that number which it cuts in the fifth circle, take halfe: then set the Index at that halfe; and it will shew in the fourth circle, the side, or root sought for.

But you must know that if the number which is the Quadrat proposed, have onely two places of Integers, the side, or root consisteth of one figure. But if it have more places of Integers, dividing them by 2, the quotient will give the true number of figures in the root, if it measure it exactly; or one lesse then the true number if any thing remaine.

Example I. The side, or root of the Quadrat 49 is required.

Set the Index at 49 in the fourth circle, and it will cut in the fifth circle 6902; in leede 1. 6902 having the *gradual number* 1 prefixed, becau'e 1 in the fourth circle signifieth 10, one *circumcision* thereof being finished: the halfe whereof is 0. 8451. Then set the Index at 0. 8451, in the fifth circle; and it will cut in the fourth circle 7, the side, or root sought for.

Example II. The side, or root of the quadrat 3249 is required.

Set the Index at 3249 in the fourth circle, and it will cut in the fifth circle 5118; indeede 3.5118 prefixing the *gradual number* 3, because 1 in the fourth circle signifieth 1000, three *circumcisions* thereof being finished: the halfe whereof is 1.7559. Then set the Index at 7559, omitting the prefixed *gradual number* 1; and it will shew in the fourth circle 57 the side sought for, consisting of two figures, because 1 in the fourth circle signifieth 10.

Example

Example III. The side of the quadrat 329476 is required.

Set the Index at 329476 in the fourth circle, and it will cut in the fifth circle 5178; indeede 5.5178 prefixing the *gradual number* 5, because 1 in the fourth circle signifieth 100000, five *circumitions* thereof being past over; the halfe whereof is 2.7589; Then set the Index at 7589, omitting the *gradual number* 2 prefixed thereto; and it will shew in the fourth circle 574 the side, or root sought for, consisting of three figures, because 1 in the fourth circle doth signifie 100.

5 The Extraction of the Cubic roots, or side is done by helpe of the fifth circle after this manner.

Set the Index at the Cube proposed; and that number which it cuts in the fifth circle divide by 3. Then set the Index at that third part, and it will shew in the fourth circle the side, or roots sought for.

And you must know that if the Cube proposed have onely three places of Integers, the side, or roote thereof consisteth of one figure: But if it have more places of Integers; divide them by 3, the quotient will give the true number of the figures of the Root, if it measure it exactly; or one lesse then the true number if any thing remaine.

Example I. The side, or roote of the Cube 343 is required.

Set the Index at 343 in the fourth circle, and it will cut in the fifth circle 5353; indeede 2.5353 prefixing the *gradual number* 2, because 1 in the fourth doth signifie 100, two *circumitions* thereof being past over: the third part whereof is 8451. Then set the Index at 8451, and it will shew in the fourth circle 7, the side sought for.

Example

Example II. The side, or roote of the Cube 185193 is required.

Set the Index at 185193 in the fourth circle; and it will cut in the fifth circle 2677; indeede 5.2677 prefixing the *gradual number* 5, became 1 in the fourth circle doth signifie 100000, five *circumitions* thereof being past over: the third part whereof is 1.7599. Then set the Index at 7599, omitting the prefixed *gradual number* 1; and in the fourth circle it will shew 57, the side, or root sought for, consisting of two figures, because 1 in the fourth circle doth signifie 10.

Examples of greater Cubes it will be needlesse to set downe.

F

CHAP.

CHAP. VI.

Of Duplicated, and Triplicated proportion.

And first of Duplicated proportion.

Theoreme.



Like plaines are in a Duplicated ratio, that is, As the Quadrats of their homologal sides. And therefore questions in the which the sides of like plaines are compared, doe appertaine vnto this place.

And it is to be noted, that if three numbers be given, in which *As the quadrat of the first, is unto the quadrat of the second; so ought the third to be unto a number sought for.* Let it be thus done, *As the first number, is to the second; so is the third to a fourth;* And againe *As the first number, is to the second; so is the fourth now found, to the number sought for.*

Example I. There are two like rectangle *Area*, or plaines, the length of the greater, is 40 feete, the length of the lesser 24 feet: each of them paved with paving tiles; the greater hath 1200 tiles: how many shall the lesser have?

The *Area*, or plaines are one to the other, as the quadrats of the longitudes given. And the proportion is direct. Say therefore

$$1600 (Q: 40) . 576 (Q: 24) :: 1200 . 432 .$$

which is the number of tiles contained in the pavement of the lesser *Area*, or plaine.

Example II. How many Acres of woodland measured with a Perch, of 18 feete, are there in 73 Acres of champane land, measured with a Perch of 16 $\frac{1}{2}$ feete?

The

The measures given (18, 16;) being reduced into their left termes, are as 12 unto 11: and the quadrats of these numbers, are, 144, and 121. And the Proportion is Reciprocall. Say therefore

$$144 (Q: 12) \cdot 121 (Q: 11) :: 73 \cdot \frac{642}{144} \quad 61 \frac{22}{144}$$

and so many are the Acres of Wood land.

Of Triplicated proportion.

3 *Theor.* Like Solids are in a Triplicated ratio, that is, As the Cubes of their homologal sides. And therefore questions in which the sides of like solids are compared, doe appertaine unto this place.

4 If three numbers be given in the which, As the Cube of the first is to the Cube of the second; so is the third number to a number sought for. Let it be thus done, As the first number is to the second; so is the third to a fourth: Again, As the first is to the second; so is the fourth now found unto a fifth. And thirdly, As the first is to the second; so is that fifth to the number sought for.

Example. If $\frac{41}{196}$ lb. of gunpowder, suffice to charge a Gun, whereof the concave diameter is inch $1\frac{1}{2}$. How many pounds of powder will suffice to charge a Gunne, whole concave diameter is 7 inches?

The capacities are one to another, as the Cubes of the diameters. And the proportion is direct. Say therefore

$31\frac{1}{2} (C: 1\frac{1}{2}) \cdot 343 (C: 7) :: 0\frac{41}{196} \cdot 43\frac{1}{2}$
wherefore $43\frac{1}{2}$ lb. of Gunpowder, will bee needfull to be had.

Another example, 43¹⁷_{lib.} of Gunpowder are sufficient to charge a Gun, whose diameter in the concave is 7 inches: now there is another sort of Gunpowder, much more strong and forcible, that is in strength unto the former, as 5 vnto 3: How much of this stronger powder, will suffice to charge a Gun of 4 inches diameter?

Here are two operations: the first seekes out, how much of that stronger powder sufficeth to charge a Gun of 7 inches diameter: and the proportion is reciprocal, that is

$$5 : 3 :: 4317 : 17148$$

The second operation is like that in the former example.


$$343 (C: 7) 64 (C: 4) :: 17148 : 3084$$

$$17148 \cdot 3126 +$$

CHAP.

CHAP. VII.

Concerning the Measuring of Circles,
Cones, Cylinders, and Sphaeres.

 *Archimedes* in a peculiar Treatise found the proportion of the Diameter of a circle to the Circumference to bee a very small deale greater then 7 unto 22: And of late *Ludolph Van Ceulen* insittling in the same steps of *Archimedes*, hath more precisely found it to be of 1 vnto 3.141592653589793 but for our Instrument it will be sufficient to take the ratio of 1 vnto 3.1416, Or of 0.183 + unto 1: leaving the diligent practizer, to more exactnesse, if he please to use his Pen.

And note that the Rules following are set downe in proportions, to be wrought as hath been taught in Chap. 2, Sect. 3. Wherein

D, or Diam. signifieth the Diameter.

Dq, or Q. Diam. the Quadrat of the Diameter.

Dc, or C. Diam. the Cube of the Diameter.

R, or Rad. the Radius, or Semidiameter.

P, or Perif. the Periferie, or Circumference.

Pq, the Quadrat of the Periferie.

Long. the length.

L the side, or latus

Alt. the altitude.

•, sheweth that the two magnitudes betweene which it is set, are to be multiplied together.

In a Circle.

2. The Diameter of a circle being given,
to finde the Periferia. Say,

$$7 \cdot 22, \text{ Or } 1 \cdot 3 \overline{1416} :: \text{Diam} \cdot \text{Perif} :$$

Example. A circle is given, the Diameter whereof is 12, I would know the circumference, or Periferia of it. Say,

$$1 \cdot 3 \overline{1416} :: 12 \cdot 37 \overline{6992} .$$

- 3 The Periferia of a circle being given,
to finde the Diameter. Say,

$$22 \cdot 7, \text{ Or } 1 \cdot 0 \overline{1183} + :: \text{Perif} \cdot \text{Diam} .$$

- 4 The Diameter of a circle being given,
to finde the Area. Say,

$$7 \cdot 4 \cdot 22, \text{ Or } 1 \cdot 0 \overline{7854} :: Q \cdot \text{Diam} \cdot \text{Area} .$$

$$\text{Or els } 1 \cdot 3 \overline{1416} :: Q \cdot \text{Rad} \cdot \text{Area} .$$

Example. A circle is given, the diameter whereof is 12, I would know the content, or Area of it. Say,

$$1 \cdot 0 \overline{7854} :: 144 (Q \cdot 12) \cdot 113 \overline{0976} .$$

$$\text{Or } 1 \cdot 3 \overline{1416} :: 36 (Q \cdot 6) \cdot 113 \overline{0976} .$$

- 5 The Area of a circle being given, to finde
the Diameter. Say,

$$22 \cdot 7 \cdot 4, \text{ Or } 1 \cdot 1 \overline{27124} :: \text{Area} \cdot Q \cdot \text{Diam} .$$

Example. A circle is given, the content whereof is 113 $\overline{0976}$, I would know the Diameter of it. Say,

$$1 \cdot 1 \overline{27124} :: 113 \overline{0976} \cdot 144 .$$

6 *The Periferia of a circle being giuen,
to finde the Area. Say,*

$$22 \cdot 4 \cdot 7, \text{ Or } 1 \cdot 0 \overline{295775} :: Q; \text{Perif} \cdot \text{Area} \cdot$$

7 *The Area of a circle being giuen, to finde
the Periferia. Say,*

$$7 \cdot 22 \cdot 4, \text{ Or } 1 \cdot 12 \overline{156637} :: \text{Area} \cdot Q; \text{Perif} \cdot$$

In a Cone.

8 *The side of a right Cone, and the Dia-
meter of the base being giuen, to find
the Superficies. Say,*

$$7 \cdot 22, \text{ Or } 1 \cdot 3 \overline{1416} :: \frac{1}{2} D \cdot L \cdot \text{Superf} \cdot$$

Example. A Cone is giuen, whereof the side is 18, and the diameter of the base 12, I would know the superficies of it. Say,

$$1 \cdot 3 \overline{1416} :: 108 (\frac{1}{2} D \cdot L) \cdot 33 \overline{2918} \cdot$$

9 *The Axis, or height of a right Cone,
and the Diameter of the base, being
giuen, to find the Soliditie. Say,*

$$7 \cdot 4 \cdot 22, \text{ Or } 1 \cdot 0 \overline{7854} :: Dq \text{ in } \frac{1}{3} \text{ Axis} \cdot \text{Soliditie} \cdot$$

Example. A Cone is giuen, whereof the Axis is 18, and the Diameter of the base 12, I would know the Solidity. Say,

$$1 \cdot 0 \overline{7854} :: 864 (Dq \text{ in } \frac{1}{3} \text{ axis}) \cdot 678 \overline{2856} \cdot$$

In

In a Cylinder.

- 10 *The side of a right Cylinder and the Diameter being given, to finde the Superficies. Say,*

$$7 \cdot 22, \text{ Or } 1 \cdot 31^{1416} :: \text{Diam} : \text{axem} \cdot \text{Superfic.}$$

- 11 *The Side of a right Cilinder, and the Diameter being given, to finde the Soliditie. Say,*

$$7 \cdot 4 \cdot 22, \text{ Or } 1 \cdot 07^{814} :: \text{Dq} \cdot \text{L} \cdot \text{Soliditie.}$$

- 12 *The Side of a right Cylinder, and the Circumference P, being given, to finde the Soliditie. Say,*

$$22 \cdot 4 \cdot 7, \text{ Or } 1 \cdot 010791775 :: \text{Pq} \cdot \text{L} \cdot \text{Solidit.}$$

In a Sphere.

- 13 *The Axis, or Diameter of a Sphere being given, to finde the Superficies. Say,*

$$7 \cdot 22, \text{ Or } 1 \cdot 31^{1416} :: \text{Dq} \cdot \text{Superficies:}$$

- 14 *The Superficies of a Sphere being given, to finde the Axis. Say,*

$$22 \cdot 7, \text{ Or } 1 \cdot 0131831 :: \text{Superfic} \cdot \text{Dq} \cdot$$

- 15 *The Segment of a Sphere being given, to finde the Superficies. Say,*

$$7 \cdot 22, \text{ Or } 1 \cdot 31^{1416} :: \left. \begin{array}{l} \text{Q: chord} \\ \text{of } \frac{1}{2} \text{ Segm.} \end{array} \right\} \cdot \text{Superfic} \cdot$$

16 *The Axis, or Diameter of a Sphere being given, to finde the Soliditie.*
Say,

$$7^{\cdot 6} \cdot 22, \text{ Or } 1 \cdot 0\text{,}\text{,}\text{,}1236 :: \text{Dc} \cdot \text{Soliditie} \cdot$$

Example. A Sphere is given, whereof the Axis is 12, I would know the Solidity of it. Say,

$$1 \cdot 0\text{,}\text{,}\text{,}1236 :: 1728 \text{ (Dc)} \cdot 597^{\cdot 108} \cdot$$

17 *The Soliditie of a Sphere being given, to finde the Axis.* Say,

$$22 \cdot 7^{\cdot 6}, \text{ Or } 1 \cdot 1\text{,}\text{,}\text{,}90986 :: \text{Soliditie} \cdot \text{Dc} \cdot$$

Example. A Sphere is given, the Solidity whereof is 5901⁶¹⁰⁸, I would know the Axis thereof. Say,

$$1 \cdot 1\text{,}\text{,}\text{,}90986 :: 5901^{\cdot 6108} \cdot 1728 \text{ (Dc)} \cdot$$

18 *A Segment of a Sphere being given to finde the Soliditie.* Say,

First. As the altitude of the other Segment, is to the altitude of the Segment given: so is that altitude of the other Segment increased by halfe the Axis, unto a fourth. Then againe say, As 7^{· 3} is to 22, Or as 1 is to 10472: so is the product of the quadrat of halfe the chord of the Perferia of that Segment, multiplied by that fourth, to the Soliditie. Viz.

$$7^{\cdot 3} \cdot 22, \text{ Or } 1 \cdot 110472 :: \left. \begin{array}{l} Q: \text{ chorda} \\ \text{in quartam} \end{array} \right\} \cdot \text{Solidit.}$$

19 For note that a Sphere, is equall to two Cones, having their height and the diameter of their base, the same with the Axis of the Sphere. Or which is all one, A Sphere is two third parts, of a Cylinder, having the height and the diameter of the base the same with the Axis of the Sphere.

CHAP. VIII.

Concerning *Plaine*, and *Solide Measures*.

He dividing of the *Carpenters ruler* into Inches, and halfe, and quarters, and halfe quarters of Inches, that is of every Inch into eight parts, is most inartificiall, and vnfit for measuring, by reason of the manifold denominations, which must be brought into one, and is hard to bee done of them that are vnskillfull. I would wish therefore that euery Inch were diuided into 10 parts, or rather that the foot were diuided into 100 parts, which is best of all: for then there will neede no reduction. And all other diuisions must bee reduced vnto this, by these Rules following.

2. If the measures be taken vpon a Ruler diuided into Inches and tenth parts of an Inch, first take out all the whole feet, and then diuide the Inches remaining, with their decimall parts if there be any by 12.

Example. How many feet and decimall parts of a foot, are in Inches 87¹?

First take out the whole foot which is 12 Inches, and there will remaine Inches 5¹: which being diuided by 12, you shall haue 443 thousand parts almost: wherefore Inches 87¹, is feet 1⁴⁴³. — . And contrariwise, feet 1⁴⁴³ —, shall be reduced into Inches 17¹, being multiplied by 12.

5. If the measure bee taken vpon a Ruler diuided into inches and halfe quarters, that is each inch into 8 parts, First you must reduce the eight parts into decimals of Inches, by diuiding the number of parts given by 8 the Denominator thereof: and afterward by the former Rule,

Rule, reduce the Inches, and decimal parts, into decimal parts of a foot.

Example. How many feet, and decimal parts of a foot, are in 7 inches, and 5 eight parts?

First diuide the 5 eight parts by 8, and you shall haue 625 thousand parts: which being put to 7 inches, will make inches 7⁶²⁵. Again diuide these by 12, as was shewed in the former rule: and the whole measure will be feete 0⁶³⁵. And contrariwise feet 0⁶³⁵, will be reduced into inches 7⁶²⁵, being multiplied by 12.

4. I must aduise all those that haue occasion, to measure Plaines, or Solids, to make themselves very perfect in this kind of Reduction (because most Rulers they shall ordinarily meet withall, are diuided into inches, and halfe quarters) which will be very easie to them, if they doe but remember, that *in diuision the first terme of the proportion implied, is the Diuisor it selfe: but in Multiplication, the first terme is euermore 1:* as hath beene shewed in Chap: 2, Sect: 7. And therefore, presuming on the diligence of the Practiser herein, I shall not neede in this kind of measuring, to speake any more of inches, but of feet and decimal parts of feet, as if the Ruler were so diuided.

Of Plaine measures.

5. A *Parallelogram*, or foure sided rect-angle Superficies, being proposed, to find the length of a Superficial foote.

Take with your Ruler the breadth thereof in feet, and decimals of a foot: and by the breadth so taken diuide 1. the quotient shall be the length of a superficial foot.

Example. A boord is feet 11⁷ broad, how much thereof will make a foot?

Divide 1 by $1\frac{1}{2}$, the quotient will bee $0\frac{2}{3}$ almost: so much shall the length of a foot be, which multiplying the parts by 12, will giue inches $10\frac{2}{3}$. And againe those parts multiplied by 8, will giue 2 eight parts of an inch.

Example, II. In tiling, or healing they vse to reckon by the Square, which is 10 foot every way, in all 100 feet. There is roose, feet $16\frac{1}{2}$ broad, how much thereof maketh a Square?

Divide 100 by $16\frac{1}{2}$ the quotient will bee $6\frac{1}{4}$ almost: so much shall be the length of one square; which multiplying the parts by 12, will be 6 feet, and inches $11\frac{3}{4}$ — almost. And againe those parts multiplied by 8, will giue somewhat more then 6 eight parts of an inch.

Example, III. In pauing they vse to reckon by the yard, which is 3 feet every way, in all 9 feet. There is a roome to bee paued, which is feet $17\frac{1}{2}$ broad; how much thereof maketh a yard?

Divide 9 by $17\frac{1}{2}$, the quotient will be $0\frac{1}{2}$ almost, the length of one yard.

6: *A fower sided recttangle Superficies,*
with all the opposite sides parallel
being proposed, to find the con-
tent.

Take with your Ruler both the breadth and length of Δ and multiply the one number into the other.

Example, A board is feet $11\frac{1}{2}$ broad, and feet $16\frac{1}{2}$ long: how many feet doth it containe in all?

Multiply $16\frac{1}{2}$ by $11\frac{1}{2}$ the product will be feete $191\frac{1}{4}$ almost: the whole quantity of that board.

Example, II. A certaine barne tiled, hath the breadth of

of the rooffe feet, $16\frac{25}{2}$, and the length of the barne is feet 47, how many Iquares of tiling hath it?

Double the length (that you may haue both sides of the rooffe) and it will be $94\frac{1}{2}$, which being multiplied by $16\frac{25}{2}$, will giue feet $1527\frac{1}{2}$. Againe diuide those feet by 100, so shall you haue Iquares $152\frac{75}{2}$.

Example. III. A certaine hall paved hath the breadth feet $17\frac{11}{2}$, and the length feet $30\frac{1}{2}$, how many yards doth it containe?

Multiply $17\frac{11}{2}$ by $30\frac{1}{2}$, the product will be feet $229\frac{11}{2}$. Diuide these by 9 and the quotient will bee yards $25\frac{1}{2}$.

7. *A fower sided Superficies with the two sides of length onely parallel being proposed, to find the contents thereof.*

Take with your Ruler the length of the two parallell sides thereof: adde both those numbers together, multiply halfe that summe, by the breadth of the Superficies taken the nearest way over, and the product will bee the content thereof.

Example. A *Trapezium*, or fower sided figure is proposed, hauing two sides thereof parallell, the length of the longer parallel side is feete $18\frac{1}{2}$ and the length of the shorter side is feete $14\frac{1}{2}$, the breadth thereof being taken the nearest way over is feete $12\frac{1}{2}$, I would know how many feete are contained in the whole Superficies?

The length of the parallell sides are feet $18\frac{1}{2}$, and $14\frac{1}{2}$, which added together make $33\frac{1}{2}$, halfe whereof is $16\frac{1}{4}$, which multiplied by the breadth $12\frac{1}{2}$, the product will be $207\frac{1}{4}$, so many feet are contained in the superficies of the Trapezium proposed.

8. A fower sided Superficiis which hath none of the sides parallel, as also euery plaine figure of more sides the fower being proposed, mult with *Diagoniall* lines bee diuided into triangles. And note that euery such figure containeth so many triangles as it hath sides, abating two out of the number. Then those triangles are to be measured seuerally as followeth.

9 To find the content, or Area of a Triangle.

Take the perpendicular height, or neereft distance betweene the base or knowne side, and the angle opposite: and by that height multiply halfe the base, or multiply the whole base by halfe that perpendicular height: and the product shall be the content, of that Triangle.

But if it be an *Equilateral triangle*: say,

1000 . 433⁰¹²⁷ :: the side of the triangle . Area.

10. To find the content of a segment of a Circle, whereof the Periferia is given in degrees and decimal parts.

First say, As 100000, is to 1745³¹⁰²⁷: so is the Arch in degrees, to the Arch in the divisions of the Radius. keepe this number found.

Againe by the 6 Sect: 1 chap: find out the true *sine* of the Arch given. Then take the difference of these two numbers found, by subducting the *Sine* out of the Arch. And lastly multiply halfe that difference by the Radius 100000, the product shall be the content of that Segment.

11. The

- 11 *The chord of any arch, together with the Radius, or semidiameter of the whole circle being given, to finde out the Arch it selfe. Say,*

As the Radius given, is to halfe the chord (reckoned in the fourth circle) so is 100000, to the sinns of halfe the arch (to be reckoned in the first, or eighth circle). Wherefore double the arch found, and so haue you the arch of the chord proposed.

- 12 *To finde a Quadrat, or Square equall to a superficies given.*

First, seeke out (as hath beene taught) the content of that superficies: then take the quadrat roote thereof by Chap. 5, Sect. 4.

Of Solid measures.

- 13 *In a Columnne, or Cyfnder, having the base, so finde how much of it maketh a foot solid.*

By a *Columnne* I meane a solid body arising from a plaine base, the angular lines whereof are parallell, and equall: and if the angular lines make right angles with the base, it is a *right Columnne*, and the length is the height thereof: but if they make oblique angles, it is an *Oblique Columnne*, and the length is not the height, but the height is a perpendicular line let downe from the top of the Columnne vnto the base, extended if needs bee: as in the
Diagramme,

Diagrame, the Solid $ABCDEFGH$, is an *Oblique Column*, because the angular line EB , standeth obliquely vpon the side of the base EA , and indeede vpon the base it selfe. Wherefore the *height* of it shall be equall to the line FP , let fall from the top vnto the base extended.



And after this manner also a *Cylinder*, and a *Pyramid*, and a *Cone* is esteemed either *right*, or *oblique*, and the *height* taken accordingly.

First therefore the base is to be found, of what fashion toer it is, as hath euen now beene shewed, either in this Chapter, or in the last before: and then diuide 1 by that same base: the quotient shall bee, the height of a Section thereof, which is equall to one foot solid.

Example. A Colunne, or peece of tymber, whose sides are all parallel, hath the breadth feet $1\frac{7}{8}$, and the thicknesse thereof is feet $1\frac{2}{3}$: which multiplied together the product will be $2\frac{2}{3}$. Diuide therefore 1, by $2\frac{2}{3}$: and the quotient shall be $0\frac{3}{4}$ almost. And so much is the height of a solid foot, of that peece of timber.

14. *Having the Base, and the height of a Colunne, or Cylinder, to finde the whole content.*

Multiply the base into the height, and the product shall be the content.

Example. A Colunne hath the base feet $2\frac{2}{3}$, and the height thereof is feet $1\frac{7}{8}$, how many feet are contained in the whole?

Multiply

Multiply the base $2,1875$, by $17,14$ and the product will be $36,986875$, so many solid feet are contained in that Columnne.

And in this very manner may you finde the content of a Cylinder, having either the diameter, or circumference given, together with the height.

15 To measure tapering timber, the base, or bases thereof, together with the height being given.

A Tapering peece of timber, according as the base thereof is right lined, or circular, is either a *Pyramide* or a *Cone*, or else a *segment* of one of these two: If it be a compleat *Pyramide*, or *Cone*, it hath but one base, Multiply that base by $\frac{1}{3}$ of the height, and the product shall be the content.

But if it be the segment of a *Pyramide*, or *Cone*, First finde out the bases at both the ends, and multiply the one by the other, and out of the product thereof extract the *Quadrat root*: then adde together both the bases, and that *quadrat root*, and multiply the Aggregate thereof by $\frac{1}{3}$ of the height, the product shall be the content.

Example. There is a tapering peece of timber, the height whereof is feet $12,6$, and the breadth of the base at the greater end is feet $11,75$, and the thicknes is feet $1,32$, which multiplied together, the product will be feet $2,31$ for the greater base: the breadth of the base at the lesser end is feet $1,2$, and the thicknesse there, is feete $0,21$, which multiplied together, the product will be feet $1,62$ for the lesser base. Multiply the bases together, the product will be $2,3252$, the *quadrat root* whereof is $1,5288$ almost, to which if you adde the summes of both the bases, the aggregate will be $4,22$, which being multiplied by $\frac{1}{3}$ of the height, *scil.* $4,2$, the product will be feet $20,958$, the content of the peece of timber.

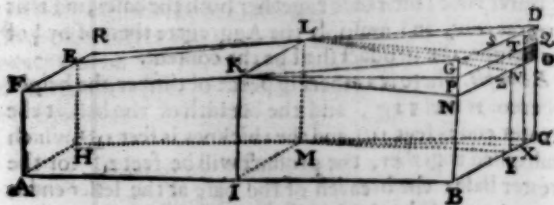
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16 Where-

16 Wherefore that vulgar manner which *Carpenters* use in measuring of tapering timber, is not true: for if a peece of timber be tapering, they measure it in the very middle, and take the base, or Section there, multiplying it by the whole length. Which this manner of working, I say, is erroneous.

For first by practise a content will be given, ever lesse then the true content found according to the former Sect: which way of working is intalibly true, as is *Analitically* demonstrated, in my *Clavis mathematica*, Cha. 20, Sect: 15.

And secondly I say, that the product of that middle base, or Section, multiplied by the length, shall bee kisse then the true content, by foure Pyramides, having for their bases, a rectangle vnder halfe the difference of the thickenesse at the ends, and a quarter of the difference of the breadths: and are as long as halfe the peece of timber: Or which is all one, by a Parallelepipedon, vnder halfe the differences of the breadths, and thickenesses, at both ends, and a third part of the whole length.



Which that I may shew, suppose one quarter of a tapering peece of timber given, sawed in linner, at halfe the breadth, and halfe the thickenesse be *ABCDGFEH*: the middle Section is *IKLM*. Measure vpon the greater base *BN*, and *CO*, equall to *AF*, and *BT*, and *NZ*, equall to *FE*: Divide *CT*, and *DO* in the mi. ft. in the points *X*, and *Q*, and draw the lines *PQ*, *NO* parallel to *BC*, and

an. YZ , JX parallel to BG . Measure also FER , equall to GS , or KL . And to the Parallelogram $BPTX$, shall be equall to the middle Section $IKLM$. Lastly, draw the lines LO , LV , LZ , and KP , KN , and MX , MT .

I say that in this one quarter of the peece of timber, A solid having the bases equall to $IKLM$, and the length AB (which is the vtuall measuring of Carpenters) is lesse then the true content, by the Pyramide $DSVOL$, in which DO , or SV , is halfe the difference of thicknes, and DS , or VO , one quarter of the difference of breadths in both ends: and the height of it equall to halfe the length.

For the two solides $AFEHIKLM$, $BNZTIKLM$, are apparantly equal, againe the two solide wedges $GSTPKL$, $PTVNKL$ are equall, and also the two solide wedges $OCXYLM$, $VXYZLM$ are equall. Now if you turne over the wedge $GSTPKL$, vnto the lesser part $AFEHIKLM$, it will over-reach in thicknesse, at the lesser end the quantity of ER , and if you also turne over the wedge $OCXYLM$, vnto the lesser part, you shall find it to fill vp the former over-reaching, and to make an exact Parallelepipedon, the bases or ends whereof are equall to the middle Section $IKLM$. but over and aboue those two wedges turned over, you shall haue left the Pyramide $DSVOL$, which was to be proved.

And in like manner if a round tapering solide, or peece of timber, be measured by the middle Section, or circular base, I say, That the product thereof multiplied by the length, shall be lesse then the true content, by a Cylinder, the diameter of whose base, is equall to halfe the difference of the diameters of the two bases: and the length is one third part of the whole length.

CHAP. IX.

Concerning the Measuring, or Gauging
of Vessells.

Wine, or Beere vessell, whether Pipe, Hoghead, Barrell, Kilderkin, or Firkin, and such like, is in forme of a *Spheroides*, having the two ends equally cut off: and accordingly may be measured thus.

Measure the two diameters of the Vessell, in inches, or else in tenth parts of a foote, the one at the bung hole, the other at the head, and also the length within. And by the diameters found, finde out the circles; then adde together two third parts, of the greater circle, and one third part of the lesse: Lastly, multiply the aggregate by the length: so shall you haue the content of the Vessell, either in Cubic inches, or cubic tenth parts of afoote.

Exempl. Suppose a vessell, having the Diameter at the bung 32 inches, and at the head 18 inches, and the length 40 inches. The quadrat of 32 is 1024. and the quadrat of 18 is 324: Say then euermore,

1. 0.¹²³⁶ :: 1024 . 536.¹⁶⁶, $\frac{2}{3}$ the circle at the bung.

2. 0.²⁶¹⁸ :: 324 . 84.⁸¹³, $\frac{1}{3}$ circle at the head.

Or else by Chap. 7, Sect. 4.

The aggregate of those two circles is 620.⁹⁷⁹, which being multiplied by 40, the length giueth 24839.¹⁶ cubic inches for the whole content of that vessell.

M. Edm.

2: Mr. Edm: Gunter in his second booke of the Crosse Staffe, Chap. 4. pretending to shew the manner of gauging Wine vessels, beginneth with these words.

"The Vessels which are heere measured, are supposed to bee Cylinders, or reduced into Cylinders by taking the meane, betweene the Diameter at the head, and the Diameter at the bottome, after the usuall manner.

And according to this supposition, teacheth to finde a Gauge point, for a gallon of wine, in that his imagined Cylindricall vessell.

Because his words are cautelous, and subterfugious, wee must a little examine them; for if his way bee true, my Rule before set downe, though grounded vpon demonstration, cannot stand.

Well then, that reduction of a wine Vessell into a Cylinder, is either true, or false; if it bee true what neede those ambiguities of Vessels which are here measured: and are supposed to be, &c. and after the usuall manner? if false why is it not noted, but deliuered as a Rule to confirme an error. And what meaneth, the meane betweene the Diameter at the head, and the Diameter at the bottome? is it the meane Geometricall, or Arithmetickall, that is the meane proportionall, or that which equally differeth from both? such shifting is vnworthy an Artift.

First therefore, Let it be the meane in respect of difference, which is equall to halfe the summe of the two Diameters: I say that the Vessell cannot truly be reduced to a Cylinder by such a meane Diameter. For seeing it is most apparent that such a Vessell, is greater in the middle then at the ends, the boords, or sides thereof, shall from the middle to the ends, goe either streight, and so the Vessel shall be as it were, two equall segments of Cones, set base to base: Or else arching, and so the Vessel shall (as before I sayd, and is commonly taken for a truth) bee a Spheroides, hauing the two ends equally cut of.

If it be considered as two segments of Cones: the measure by that *mean Diameter*, or middle Section is quite false, as hath beene demonstrated in the former Chapter, Sect: 16, 17. and will be given lesse then the true content, although the sides goe straight: much more then, if the sides goe arching; for that convexnesse, must needs yeeld a greater capacity. And therefore in neither can that manner of Gauging be true.

Againe, if the *mean Diameter* be vnderstood to be the *mean proportionall* betwene the two Diameters, it is much more false, for betwene any two numbers, the *mean Geometrical* is lesse then the *mean Arithmetical*.

Thus much I haue thought good in this place ingeniously to signifie to the inexpert Learner, that hee might not beguile himselfe with a prejudged opinion.

3. I haue shewed the measuring of vessels by the cubic inch: but our vnuall reckoning is by the Gallon, and parts thereof. Wee must therefore doe the best we can, to inquire the *true quantitie* of a Gallon in inch measure, which will bee difficult to doe exactly, both because the *Standards* vsually are not streight sided, but a little arching, neither doe they agree perfectly one with another: but what partly by experience, both mine owne, and others, which hath come to my sight; and partly by reasoning shall seeme to me most probable, I will not refuse to set downe.

4. Our *English Gallon* is vnderstood to bee either in *Ale measure*, or *Wine measure*: and these two measures not a little differing. And first we will inquire about our *Ale measure*.

I my selfe haue measured Bushels, and Pecks, which haue exactly beene fitted to the *Standards*, and haue still in my account found a Gallon to containe better then 270 cubic

cubic inches, indeed much about 273, or 273 as precisely as I could measure in a vessell not truly regular. Alto my worthy friend *Master William Twiss*, who hath vndergone great paines and charge, in finding out the true content of our *English measures*, gaue vnto me two severall measures of an *Ale-gallon*, and those in due consideration but little differing. The one was found out by a brassen vessell made in manner of a Parallelepipedon, the base whereof was exactly sixe inches square, and the sides divided into inches and twentieth parts: into which vessell he powring out a *standard Gallon of Quene Elizabeth*, filled with water, found it therein to aunc vnto 7 inches, and 6 tenth parts: which being computed maketh cubic inches 273 $\frac{6}{10}$. The other was found by taking the dimensions of that *standard Gallon*, which was made in forme of a *segment of a Cone*, but that the sides were a little arching: the dimensions were thus; the Diameter of the top was inches 6 $\frac{1}{2}$: the Diameter of the bottome was inches 5 $\frac{1}{2}$: and the height of it was inches 9 $\frac{1}{2}$: which being cast vp by Chap: 8, Sect: 17, will be found to containe cubic inches 268 $\frac{85}{100}$: differing from the former, only cubic inches 4 $\frac{1}{2}$: which difference might well arise through the curuitie of the sides. These measures he did not only take himselfe, but to giue me satisfaction, shewed me the experience in the said *Vessell and Standard*: but the truth is, I obserued the *Standard*, besides the arching of the sides, to bee not exactly circular within, nor the brimme of an even height, nor the bottome plaine: and in taking the height of the water in the *Vessell*, our sight was not able to estimate the ascent thereof so precisely, that a spoonefull of water, more or lesse, could breed any sensible difference.

What therefore shall wee doe in this difficultie? indeede looke to the first ground, and principle of our *English measuring*, from Barley cornes. For the length of
3 Barley

3 Barley cornes taken out of the middle of the eare is an *Inch*, or *Uncia*, that is a twelfth part of a foot. 3 feete make a *Yard*: and 16 feet and an halfe, that is 5 yards and an halfe, a *Perch*, with which wee measure our land: for 40 perches is a *Furlong*, and 8 furlongs an *English mile*: and againe 40 square perches is a *Roodland*, 4 roodland make an *Acre*. So then a *Perch* which is feet $16\frac{1}{2}$, or yards $5\frac{1}{4}$ is as it were the beginning of all land measure in length: and a square *Perch* which is feete $272\frac{1}{4}$, is as it were the beginning of all land measure in the superficial content.

Now therefore seeing in *Vessels* a *gallon* is as it were the beginning of *Vessell measure* (for a *Pottell* is but a diminutive of it, and a *quart*, the quarter) it is not vnlikely that our wise Ancestors had such a consideration also in solide measures, that as a square *Perch* (the beginning of Superficiall land measure) did containe 272 square feete and 1 quarter; so a *Gallon* (the beginning of vessell measure) should containe 272 Cubic inches and a quarter. And the rather seeing that the ancient *Geographers*, diuide a foote into 4 *Palmes*, a *palme* being 3 of our inches, as 3 feete are a yard. So that as the side of a square *Perch* consisteth of yards $5\frac{1}{4}$, a *Gallon* also should consist, of a number of Cubic inches the square side whereof is *palmes* $5\frac{1}{4}$.

Wherefore sauing the exact truth when it shall appeare, and in the meane time the more probable reasons of other men, I make bold to tender this my coniecture, to the censures of more diligent Inquirers, That the measure of an *English Ale gallon* should be a square Vessell of inch $16\frac{1}{2}$, or *Palmes* $5\frac{1}{4}$ every Way, and 1 inch deepe: that is $272\frac{1}{4}$ Cubic inches.

5. And this my opinion may peraduenture receive some confirmation by the inquiry of an *English wine Gallon*.

Master

M. Henry Briggs that learned Geometrician, and my very loving friend, made an experiment, with a Cubicall vessell, which was 12 inches euery way, which hauing filled with water carefully measured, found it to containe 7 gallons and an halfe wanting a *moment*, as hee himselfe long since, being then of *Gresham Colledge*, signified to me. Now if it had contained exactly 7 gallons, and an halfe, a *Wine gallon* should haue beene 2304 cubic inches, but because it wanted a little, the gallon must be somewhat bigger, for which *moment* therefore if you will put 6 hundred parts of an inch, the *Wine gallon* shall containe 231 cubic inches.

Againe M. Gunter in the place before mentioned sheweth, that the common opinion is that at *London*, a *Cylindricall vessell*, whose diameter is 38 inches, and length 66 inches, doth containe 324 gallons: wherefore by this account a gallon should be 231 cubic inches almost exactly, which in both so neerely agreeing, wee may well conclude, That an *English Wine gallon* doth containe 231 cubic inches.

$$\sqrt{92914815} \text{ est } 171507$$

It is also commonly receiued, that the reason of the greatnesse of an *Ale gallon* about the *Wine gallon* is, that because of the frothing of the Ale or Beere, the quantity becommeth lesse, and therefore such liquors that did not so yeeld froth, as Wine, Oyle, and the like, should in reason haue a lesser measure. If then we compare these two gallons together, we shall finde that

$$272\frac{1}{2} \cdot 231 :: 161 \cdot 14 \cdot$$

which abatement might to our Ancestors, in apportioning those measures, seeme to be reasonable.

6 To finde how many Cubic tenth parts of a foot are in a gallon, both of Beere, and Wine : or also in any number of Cubic inches.

Because there be in a Cubic foote, 1000 Cubic tenth parts, and 1728 Cubic inches, say,

1728 (C: 12) . 1000 (C: 10) :: 273¹⁵ . 157⁵³²¹ —
cubic tenth parts of a Beere gallon.

And 1728 (C: 12) . 1000 (C: 10) :: 231 . 133⁶⁸⁰³ —
cubic tenth parts in a Wine gallon.

Also 1728 . 1000 :: 24839¹⁶ . 14374⁷⁴⁶ ,
cubic tenth parts, are in the vessell measured in Sect. 1.

7 And according to those measures so found, you may easily finde the content of a pint, or quart, or peck, or bushell, which two last are to be reckoned in Ale measure.

8 The Content of a Vessell, being given in cubic inches, or in cubic tenth parts of a foot, so finde how many gallons it containeth.

This is easily done if you diuide the content given in inches, by 273¹⁵ for Ale measure : and by 231, for Wine measure. But if the content be given in decimall parts of a foote diuide it by 157⁵³²¹ — for Ale measure; and by 133⁶⁸⁰³ — for Wine measure.

Example. How many wine gallons are in a vessell containing 24839¹⁶ cubic inches, or 14374⁷⁴⁶ cubic tenth parts of a foote. Diuide 24839¹⁶ by 231, or diuide 14374⁷⁴⁶ by 133⁶⁸, and the quotient shall be 107¹³ wine gallons.

CHAP. X.

Concerning the Comparison of sundry Metals,
in quantity and weight.



IF foure pieces of Metalls, whereof the third is of the same kinde with the first, and the fourth of the same kind with the second, are proportionall, their grauities also, or weights, shall be proportionall.

2. If there bee foure pieces of metall, whereof the third is of the same kind with the first, and the fourth of the same kinde with the second: and the first and second be of equall greatnesse, and the third and fourth of equall weight; the weights of the first and second, shall be reciprocal to the magnitudes of the third and fourth.

3. Two Sphares of the same matter are in weight, as the cubes of their Diameters, are in magnitude. *Et contra.*

4. Pieces of Metall if they bee of equall magnitude, haue their weights in direct proportion, as is here set downe: but if they be of equall weight, they haue their magnitudes in proportion reciprocal: According to the experiments of *Marinus Ghetaldi*, in his tractate called *Archimedes promissus*.

Gold.	3990	Brasse.	1890
Arg. Viu.	2850	Iron.	1680
Lead.	2415	Tinne.	1554
Silver.	2030		

5. To finde the Weight of a Sphere of Tinne, hauing the Diameter 1 Inch, or else 1 tenth part of a foot.

Take a piece of Tinne, and turne it exactly in a Lathe, into a Cylinder, hauing both the Diameters of its base, and also the length equall, to the Diameter of the Sphere giuen. Then weigh that Cylinder, that you may haue

the weight thereof in graines: And lastly, take two third parts of the whole number of graines: for the weight of the Sphere.

After this manner *Marinus Ghetaldi* found a Cylinder of 1 inch, or twelfth part of a foot thick, and long to weigh 1824 graines: whereof; is 1216, the weight of a Sphere of that thicknesse.

Againe if you say,

1000 (C: 10) . 1728 (C: 12) :: 1216 . 2101, ²⁴⁸/₁₀₀₀

You shall haue the weight of a Sphere whose Diameter is one tenth part of a foot.

Wherefore also a Cubed inch of Tinne weigheth 2322, ⁴/₁₀₀₀ —, and a Cubed tenth part of a foote weigheth 4013, ¹⁷/₁₀₀₀ —

And note that *Mar: Ghetaldi* useth the ancient Roman foot, which by the measure set downe in his booke seemeth to be very little lesse, then our vsuall English foot, if not exactly the same.

Note also that he diuideth one pound, into 12 ounces, and every ounce into 24 Scruples, and euery Scruple into 24 graines: So that an ounce with him weigheth 576 graines: and a pound 6912. Whereas our English pound of Troy Weights by *Affies*, or Goldsmiths Weights, is but 5760 graines, and our ounce 480. But whether the *Ragazean* grain, be the same with our English, I leaue to be tryed by the diligent Practizer.

6th To finde the Weights of a Sphere of Tinne, at any other Diameter assigned.

Multiply the Cube of the Diameter given by 1216, if it be in inch measure; or by 2101, ²⁴⁸/₁₀₀₀, if the measure be by decimall parts of a foot: and the product will be the weight of that Sphere.

And contrariwise to find the Diameter of a Sphere of Tinne, by the weight given in graines. Diuide the weight given in graines by 1216 if you would haue inch measure:

measure: or by 2101,²⁴ if you would measure by decimal parts of a foot, and the quotient shall be the Cube of the Diameter.

7. To finde the *Weight of a Sphere of any Metall, at any Diameter giuen, either in Inch measure, or in decimal parts of a foot.*

First by Sect. 6. seeke the weight of a Sphere of Tinne, at that Diameter: then by Sect. 4, say, *As the proportionall number of Tinne, is to the proportionall number of that other Metall: so is the Weight of the Sphere of Tinne now found, to the Weight of the Sphere proposed.*

Example. Suppose a Sphere of Iron, whose Diameter is 3 inches; what shall be the weight thereof?

First, the weight of a Sphere of Tinne, of 3 inches Diameter, will be found to be 32832 . 35494⁰⁵⁴ grains the weight of the Sphere proposed.

- 8 To finde the *Diameter of a Sphere of any Metall in inch measure, or decimal parts of a foot, the Weight thereof being giuen.*

First by the contrary of Sect. 6, seeke the Cube of the Diameter of a Sphere of Tinne of that weight. Then by Sect. 4, say, reciprocally. *As the proportionall number of that other Metall, is to the proportionall number of Tinne: so is the Cube of the Diameter now found, to the Cube of the Diameter of the Sphere proposed.*

Example. A Sphere of Iron weigheth 35494⁰⁵⁴ grains: how many inches is the Diameter thereof?

First, the Cube of the Diameter of a Sphere of Tinne of 35494⁰⁵⁴ grains weight, will be 29,¹⁸⁹¹⁰⁶⁹⁵ then say, reciprocally,

$$1680 \cdot 1554 :: 29,¹⁸⁹¹⁰⁶⁹⁵ \cdot 27$$

The Cubic root whereof is 3, the Diameter of a Sphere of Iron of that weight proposed.

CHAP. XI.

Concerning the Ordering of Souldiers, in any kinde of rectangular forme of battaile.

Battailes are considered either in respect of the number of men, or in respect of the forme of ground. *As a square battaile of men* is that which hath an equall number of men, both in *Rank* and *File*, though the ground on which they stand, bee longer on the *File*, then on the *Rank*. And a *square battaile of ground* is that which hath the *Rank* as long as the *File*, though the men in *Rank* be more then in *File*.

2. In respect of the number of men, it is called either a *square battaile*, or a *double battaile*, or a *battaile of the grand front*, which is quadruple, or a *battaile of any proportion*, of the number in *Rank*, to the number in *File*.

3. If it bee a *square battaile of men*: Extract the quadrat root out of the whole number of men, and the same shall be the number of Souldiers, to be set in a *Rank*.

Example. 576 Souldiers are to bee martialled in a square battaile, that so many may be in *Rank*, as in *File*.

Take the quadrat root of 576, which is 24: the same shall be the number to be placed in a *Rank*.

4. If it be a *double battaile of men*: Extract the quadrat root out of halfe the number of men, and the same doubled shall bee the number of Souldiers to bee set in a *Rank*.

Example. 1458 Souldiers, are to be placed in a double battaile; so that twise so many may be in *Rank* as in *File*.

Take

Take halfe the given number 1458, which is 729, the quadrat root whereof is 27: double it and you shall haue 54 men to be placed in a ranke.

5. If it be a *quadruple battaile*, which is called of the *great front*: Extract the quadrat root out of one quarter of the number of men, and the same quadrupled shall be the number of Souldiers to be set in a ranke.

Example. 1024 Souldiers are to be martialled into a *battaile of the grand front*, so that fower times so many be in ranke as in file.

Take one quarter of 1024 the number given, which is 256, the quadrat root whereof is 16: quadruple it, and you shall haue 64 men to be placed in a ranke.

6. If a *battaile* be required of any other forme, that is, if a Ratio be given, according to which the number of men in Ranke, shall be to the number in file. Multiply the two termes of the Ratio given: Then say *As the product is to the quadrat of the terme which is for the ranke, or As the terme which is for the file, is to the terme which is for the ranke: so is the whole number of Souldiers, to the quadrat of the number of men to be placed in a ranke.*

Example. 1944 Souldiers, are to be martialled so, that the number of the ranke, be to the number of the file, as 8 vnto 3, that is for 8 men in ranke, 3 are to be set in file.

First multiply the two termes of the Ratio 8 and 3, the product whereof is 24, also quadrat 8, the terme of the ranke, which will be 64. Then say,

$$3 \cdot 8 :: 1944 \cdot 5184$$

out of which extract the quadrat root 72, and it will giue you the true number of the ranke.

7. In respect of the forme of ground, the *battaile* is either

either a *square of ground*, or longer one way then the other. For the distance, or order of Souldiers martialled in array, is distinguished either into *Open order*, or *Order*.

Open order is when the very centers of their places, are distant 7 feet asunder, both in ranke and file.

Order is when the centers of their places, are distant 3 feet and a halfe in *ranke*, and so much in *file*. Or else 3 feet and a halfe in *ranke*, and 7 feet in *file*: which last *order*, and whatsoever *order* else there is, in which the distance of the *rankes* one from another is greater, then the distance of *files*, causeth that a *square of men*, maketh not a *square of ground*, but the ground is longer on the *file* then on the *ranke*.

8 If it be a *square battaile of ground*, the centers of the distances being feet $3\frac{1}{2}$ in *ranke*, and 7 feet in *file*. Because $3\frac{1}{2}$ is halfe of 7, the ratio of the distances, is as 1 unto 2. And seeing the number in ranke, to the number in file, is reciprocal to the distances, the ratio of the number of men in ranke, to the number of men in file, shall be as 2 unto 1. And so the Rule shall be the same with that in Sect. 6, namely, *As the terme of the file, is to the terme of the ranke; so is the whole number of Souldiers, to the true number of the ranke.*

Example, 1352 Souldiers, are to bee set in a *square of ground*, that their distances may be feet $3\frac{1}{2}$ in ranke, and 7 feet in file.

The Ratio of the *ranke* to the *file*, shall reciprocally be, as 7 to $3\frac{1}{2}$, that is as 2 to 1. Say therefore

$$1 \cdot 2 :: 1352 \cdot 2704,$$

the quadrat roote whereof 52 is the number of men to be set in a ranke.

9 If a battaile wherein the distance in ranke is vnequall to that in file, be longer one way then the other, according

ding to any Ratio giuen : there is to be considered a double ratio, one reciprocal in respect of the distances, the other according to the forme of the ground. Wherefore to finde the Ratio of men in ranke, to the men in file, Multiply the two termes of the ranke, for the ranke, and the two termes of the file, for the file. And then the Rule shall bee the same with that in Sect. 6, namely, *As the terme of the file, is to the terme of the ranke : so is the number of Souldiers, to the quadrat of the true number of the ranke.*

Example. 10290 souldiers, are to be set in a battaile, so that they may stand onely 3 feet asunder in ranke, and 7 feet in file, and the length of the ground for the ranke, to the length of the ground for the file, shall haue the ratio of 5 vnto 2.

First in respect of the distances, the Ratio of Rank to file, reciprocally is as 7 vnto 3. Secondly, in respect of the ground, the ratio of rank, to file, is as 5 to 2. Wherefore by multiplication of like termes, the true ratio of rank to file shall be 7×5 to 3×2 , that is as 35 to 6. Say therefore

$$6 \cdot 35 :: 10290 \cdot 60035$$

the quadrat root whereof is 245, the number of men to be set in ranke.

10 If 1000 Souldiers, may be lodged in a square, of 300 feete, how many feete must the side of a square be, which will serue to lodge 5000? Say,

$$1000 \cdot 5000 :: 300 \cdot 300 \cdot 450000$$

the quadrat roote whereof 671 — is the square side sought for.

And this is the order for resolution of all other questions of this sort.

CHAP. XII.

*A collection of the most necessarie
Astronomicall operations,*

BEfore wee deliuer the Rules of such operations, it will not be inconuenient, to set downe certaine *Reductions*, whereof we may haue frequent vse.

To reduce sexagesime parts into decimals.
Diuide the sexagesimes given by 60.

Example. How many decimals are $34', 12''$?

Here are required two reductions, first of the seconds into decimals of minutes: then of the minutes with their decimals, into decimals of degrees, Thus

$$60 \cdot 1 :: 12'' \cdot 02$$

Againe

$$60 \cdot 1 :: 34.2 \cdot 057$$

Wherefore $34', 12''$ are equall to 0.57 of a degree.

And contrariwise to reduce decimal parts of degrees into sexagesimes. Multiply the decimal part given by 60.

Example. How many sexagesime parts are 0.57 ?

$$1 \cdot 60 :: 0.57 \cdot 34.2$$

Againe

$$1 \cdot 60 :: 0.2 \cdot 12''$$

To reduce houres into degrees. Multiply the houres with their decimal parts by 15.

Example

Example. How many degrees are $8^{\text{Ho}}, 34', 12''$; that is by the former reduction 8.57^{Ho} ? thus

$$1 \cdot 15 :: 8.57 \cdot 128.55$$

Wherefore Houres 8, 34', 12" doe containe 128.55 degrees.

And contrariwise to reduce degrees into houres. Diuide the degrees with their decimall parts by 15.

Example. How many houres are in degrees 128.55?

$$15 \cdot 1 :: 128.55 \cdot 8.57$$

a It is to be vnderstood, that if foure numbers are proportionall, their *Order* may be so transposed, that each of those termes, may bee the last in proportion. In this manner,

- I. As the first is to the second; so is the third to the fourth.
- II. As the third is to the fourth; so is the first to the second.
- III. As the second is to the first; so is the fourth to the third.
- IIII. As the fourth is to the third; so is the second to the first.

Wherefore euery proportion doth implicitly containe foure *Orders*, two descending, and two ascending, as may be seene by their combinations: By one of which *orders*, if of foure proportionall numbers, any three be giuen, that other which is ynnowne, may be found out.

K 2

Example

Example. To finde out any of these,

- terms { 1 As the Sine of the complement of the suns
declination,
2 is to the Sine of the compl. of his altitude;
3 So is the Sine of the Sunnes Azimuth from
the meridian,
4 to the Sine of the horary distance from the
meridian.

If the first, second, and third termes be giuen, the fourth shall be found out by the *I order*.

If the first, third, and fourth termes be giuen, the second shall be found out by the *II order*.

If the first, second, and fourth termes be giuen, the third shall be found out by the *III order*.

If the second, third, and fourth termes be giuen, the first shall be found out by the *IIII order*.

3. To finde out any one of these.

- terms { 1 As the Radius, or totall Sine
2 is to the Sine of the distance, or longitude of
the Sunne in the Ecliptic, from the next
Æquinoctial point:
3 So is the Sine of the Sunnes greatest decli-
nation (which is the angle of the Ecliptic
with the Æquinoctial),
4 To the Sine of the Sunnes declination in
that longitude.

4. To

4. To finde out any one of these.

- termes { 1 As the *Radius*,
 2 is to the Sine of the *Sunnes right ascension*,
 from the next æquinoctiall point :
 3 So is the tangent of the *Sunnes greatest de-*
clination,
 4 to the tangent of the *Sunnes declination* in
 that place.

5. To finde out any one of these.

- termes { 1 As the *Radius*
 2 is to the Sine of the compl. of the *Sunnes*
greatest declination :
 3 So is the tangent of the *longitude of the*
Sunne from the next æquinoctiall point.
 4 to the tangent of the *Right ascension of the*
Sunne, from the same æquinoctial point.

6. To finde out any one of these.

- termes { 1 As the *Radius*
 2 is to the Sine of the compl. of the *longitude*
of the Sunne from the next æquinoctiall
 point ;
 3 So is the tangent of the *Sunnes greatest de-*
clination,
 4 to the tangent of the compl. of the *angle of*
the Ecliptic with the Meridian,

7. To finde out any one of these.

- terms {
 1 As the *Radius*,
 2 is to the Sine of the *Sunnes greatest decli-*
nation;
 3 So is the Sine of the compl. of the *Sunnes*
right ascension from the next æquinoctial
 point,
 4 to the Sine of the compl. of the *Angle of*
the Ecliptic with the Meridian.

8. To finde out any one of these.

- terms {
 1 As the Sine of the compl. of the *Poles*
height,
 2 is to the *Radius*;
 3 So is the Sine of the *Sunnes declination*,
 4 to the Sine of the *Sunnes Amplitude ortive*,
 that is the arch of the horizon from the
 place of the *Sunnes rising or setting* to
 the true East, or West point.

9. To find out any one of these.

- terms {
 1 As the *Radius*,
 2 is to the Sine of the *Sunnes greatest ampli-*
tude ortive, which is in the Tropicks;
 3 So is the Sine of the *longitude of the Sunne*
 from the next æquinoctial point,
 4 to the Sine of the *Sunnes amplitude ortive*.

Or

Or also of these.

- terms { 1 As the Sine of the compl. of the Poles
height,
2 is to the Sine of the compl. of the Sunnes
greatest declination:
3 So is the Sine of the Sunnes longitude from
the next æquinoctial point,
4 to the Sine of the Sunnes amplitude or sine.

10. To finde out any one of these.

- terms { 1 As the Radius,
2 is to the tangent of the height of the Pole.
3 So is the tangent of the Sunnes declination,
4 to the Sine of the Sunnes Ascensional dif-
ference.

11. To finde out any one of these.

- terms { 1 As the Radius
2 is to the Sine of the height of the Pole:
3 So is the tangent of the Sunnes amplitude
or sine,
4 to the tangent of the Sunnes ascensional
difference,

12 To

12. To finde out any one of these.

- termes {
 1 As the Sine of the compl. of the *Sunnes*
declination,
 2 is to the *Radius* :
 3 So is the Sine of the compl. of the *Sunnes*
amplitude versine,
 4 to the Sine of the compl. of the *Sunnes*
ascensional difference.

13. To finde out any one of these.

- termes {
 1 As the tangent of the *height of the pole,*
 2 is to the *Radius* :
 3 So is the tangent of the *Sunnes declination,*
 4 to the Sine of the *Suns horary distance from*
the Meridian, being due East or West.

14. To finde out any one of these.

- termes {
 1 As the Sine of the *height of the pole*
 2 is to the *Radius* :
 3 So is the Sine of the *Sunnes declination,*
 4 to the Sine of the *Sunnes altitude being due*
East, or West.

15. To

15. To find out any one of these.

terms { 1. As the Radius,
2 is to the Sine of the height of the Pole:
3 So is the Sine of the Sunnes declination,
4 to the Sine of the Sunnes altitude above
the Horizon at sixe of the Clocke.

16. To find out any one of these.

terms { 1. As the Radius,
2 is to the Sine of the complement of the
Poles height.
3 So is the tangent of the Sunnes declinati^on.
4 to the tangent of the Sunnes Azimuth fr^om
the North meridian, at 6 of the Clocke.

17. The hower of the Sunnes Rising, and setting is found out by the Ascensionall difference. For if you reduce the degrees of the Ascensionall difference, into howers, it will shew you how much the Sunne riseth, or setteth before, or after 6 a Clocke.

18. The Oblique ascension also of the Sunne is found out by the Ascensionall difference. For if you subduct the Sunnes Ascensionall difference, out of the right ascension of the Sunne, from the beginning of Aries, for the sixe Northerne signes which are Υ , φ , Π , \S , Ω , ϖ , or if you adde it thereto, for the sixe Southerne Signes, which are \cap , m , \uparrow , ψ , ϖ , χ . you shall have the Sunnes oblique ascension.

19. The declination of the Sunne, and his Altitude above the Horizon at any time, together with the height of the Pole being giuen, to find the hower of the day. Say,

As the Radius,
is to the Sine of the complement of the Sunnes
declination.

So is the Sine of the compl. of the Poles height,
to a fourth number. **Keepe it,**

Then out of the Sunnes distance from the North pole,
subduct the complement of the Pole; and of that re-
maines, and the complement of the Sunnes altitude, take
both the Summe, and also the difference. And say againe,

As the fourth before kept,

is to the Sine of halfe that summe:

so is the Sine of halfe the difference,

unto a number which being multiplied

by the Radius, is equal to the quadrat,

of the Sine of halfe the Angle of the

Sunnes horary distance from the Meridian.

20. The declination of the Sunne, and his alti-
tude above the Horizon at any time together
with the height of the Pole being given, to find
the Sunnes Azimuth. **Say,**

As the Radius

is to the Sine of the compl. of the Sunns altitude,

So the Sine of the compl. of the Poles height,

is to a fourth, **Keepe it,**

Then out of the complement of the Sunnes alti-
tude, subduct the complement of the Pole; and of that re-
maines, and the distance of the Sunne from the North
Pole, take both the Summe, and also the difference: and
say againe,

As the *fourth before kept*,
 is to the Sine of *halfe the summe* :
 So is the Sine of *halfe the difference*,
 vnto a number which being multiplied
 by the Radius, is equall to the quadrat of
 the Sine of halfe the angle of the *Sunnes*
horizontall distance from the Meridian.

27. To find the length of the *Crepuscu-*
lum, or Twilight.

Betweene the light of the day, and the darkenesse of
 the night, the Twilight is set by the wise Creator; that
 wee here vpon the earth might not in an instant, passe
 from one extreame into another, but by successiue de-
 grees. *The Twilight is nothing else but the refraction of*
the Sunnes beames, in the densitie of the aire. And *Pet.*
Nonnius to find the length of the Twilight, watched the
 time after Sunne set, when the twilight in the West was
 shut in, so that no more light appeared there, then in any
 other part of the sky neere the Horizon: then by one
 of the known fixed Stars, having taken the true hower
 of the night, found by many obseruations, that at the
 time of shutting in the Twilight, the Sunne was vnder
 the Horizon 18 degrees, and vntill the Sunne was gone
 so low, the Twilight continued. Say therefore

As the Radius
 is to the Sine of the compl. of the *Sunnes*
declination :

So the Sine of the compl. of the *height*
of the Pole,
 is to a fourth,

Keep it.

Then out of the Sunnes distance from the South Pole,
 subduct the complement of the Pole; and of that re-

maines, and degrees 63, take both the *Summe* and also the *difference*; and say againe,

As the *fourth kept*,
is to the Sine of *halfe the summe*:
so is the Sine of *halfe the difference*,
to a number which being multiplied
by the Radius is equal to the quadrat
of the Sine of *halfe the angle of the*
Sunnes distance at the ending of the Twi-
light, from the high Noone next to it.

Wherefore if out of the whole angle converted into hower, you subduct halfe the diurnall arch, or the hower of the Sunnes setting, you shall haue the true length of the *Crepusculum*, or *Twilight*.

21. To find the length of the least *Crepusculum* in the year.

The Sunne being in the winter *Tropic* maketh the longest *Crepusculum*, of the whole winter halfe year, and from thence, as the dayes increase, the *Crepuscula* doe decrease vntill they come to bee shortest, which is in a certaine *Parallel*, betweene that *Tropic*, and the *Aequinoctiall*: the declination whereof is thus found out.

As the tangent of the complement of the Pole,
is to the Sine thereof:
So is the tangent of 9 degrees,
to the Sine of the declination of the *Parallel*,
in which the Sunne maketh the shortest *Crepusculum* of the whole year.

23 But before the *Crepusculum* come to bee shortest, there is another Parallel, in which the *Crepusculum* is equall to that in the *Equinoctiall*: the declination whereof is thus found our.

As the *Radius*

is to the Sine of the altitude of the Pole:
So is the Sine of 18 degrees
to the Sine of declination of the Parallel
in which the Sunne maketh the Twilight
equall to that in the *Equinoctiall*.

24. If an Arch of the *Ecliptic*, be equall to his Right ascension, one end thereof being knowne, to find out the other end. Say.

As the Sine of the Compl. of the declination
of the arch given.

is to the *Radius*:
so is the Sine of the compl. of the greatest
declination,
to the Sine of the compl. of the other end.

25. To find the points of one quadrans of the *Ecliptic*, wherein the difference of longitudes cease to be greater, then the differences of the right ascensions.

Multiply the Sine of the complement of the greatest declination, by the *Radius*, and out of the product extract the quadrat root: the same shall bee the Sine of the complement of the declination sought for.

26. To find the *quantities of the angles*,
Which the circles of the 12 Houses
make with the Meridian. Say

As the *Radius*

is to the *Tang.* of 60 deg. for the 11th, 9th, 5th,
 and third houses, or to the *Tang.* of 30 deg.
 for the 12th, 8th, 6th, and second houses;
 so is the *Sine* of the complement of the *Pole*,
 to the *tang.* of the *compl.* of any house with the
Meridian.

And note that on the Easterne part of the vpper hemisphere, there are *three circles of Houses*, the *Horoscope*, which is also the *Horizon*, and next to that is the *circle of the 12th House*, then the *circle of the 11th House*. On the Western part also, are *three circles of Houses*, the *circle of the 7th House*, which also is the *Horizon*, and next thereto the *circle of the 8th House*, then the *circle of the 9th House*. But the *circle of the 10th House*, is the *very vpper Meridian* itself. *Contrary Houses* are 1 and 7, 2 and 8, 3 and 9, 4 and 10, 5 and 11, 6 and 12.

27. Resolue the whole time from the Noone last past into degrees (by multiplying the *hours* with their *decimall parts* by 15, according to Sect. 1) which adde vnto the *right Ascension* of the *Sunne*: and you shall haue the *right ascension* of the point of the *Aequator* in the vpper *Meridian*, which is called the *Right ascension of Medium cali.*

28. Adde 99 degrees to the *Right ascension* of *Med. Calis* and it shall be the *degree of the Aequator* then rising upon the *East Horizon*.

29. If

29. If the *first quadrant of the Equator doe arise*, the beginning of γ is distant from the meridian Eastward, so much as is the distance of the *Right ascension of Med. cæli*, from 360. But if the *second quadrant of the Equator doe arise*, the beginning of γ , is distant from the Meridian Westward, so much as is the distance of \odot , from the *Right ascension of Med. cæli*.

And in both of them the lower angles of the Ecliptick with the Meridian, on the East side is obtuse, and on the West side acute: and the 90th degree of the Ecliptick, commonly called *nonagesimus gradus*, is on the East part.

30. If the *third quadrant of the Equator doe arise*, the beginning of α is distant from the Meridian Eastward, so much as is the distance of the *Right ascension of Med. cæli* from 180. But if the *fourth quadrant of the Equator doe arise*, the beginning of α , is distant from the Meridian Westward, so much as is the distance of 180, from the *Right ascension of Med. cæli*.

And in both of them the lower angle of the Ecliptick with the Meridian, on the East side is acute, and on the West side obtuse: and on the 90th degree is on the West part.

31. The *point of the Ecliptick culminant in the Meridian*, which is called *Medium cæli*, or *Cor cæli*, and is the *cuspis of the 12th house*, may be found by Sect. 5.

32. The *declination of the said culminant point*, may be found by Sect. 3. VVherefore also by adding or subtracting that declination, to, or from the elevation of the *Equator*,

Equator, (which is the complement of the Pole) the *Altitude of Med. cali* may be had:

33. The *Angle of the Ecliptick with the Meridian*, may be found by Sect. 7.

34. To finde the *Altitude of the 90* *degr.* Or the *Angle of the Ecliptick with the horizon.*

As the *Radius* is to the *Sine* of the compl. of the *altitude of Med. cali.*

So is the *Sine* of the *angle of the Ecliptick with the Meridian,*

to the *Sine* of the compl. of the *angle sought for.*

35. To finde the *Azumith of 90* *degr.* which is also the *Amplitude ortive of the Ascendens, or Horoscopus.*

As the *Radius,* is to the *Sine* of the *Altitude of Med. cali.*

So is the *tang.* of the *Angle of the Ecliptick with the Meridian,*

to the *tang.* of the compl. of the *distance of that Azumith from the Meridian.*

36. To finde the *Horoscopus, or Ascendent degree of the Ecliptick, Or the Cuspis of the first house.*

The *Distance* of the *Azumith of 90* *degrees* from the *Meridian,* is equal to the *Amplitude ortive* of the *Ascendent degree.* Wherefore the *Ascendent degree* of

of the Ecliptic, may thence be found, by Sect: 8, or 9.
Or else thus

As the Radius
is to the Sine of the complement of the angle
of the Ecliptic with the Meridian:

So is the tang. of the complement of the
altitude of Med. Celi,
to the tangent of the distance of Med. celi
from the Ascendent degree.

37. To find the parts of the angle of the Ecliptic with the Meridian, cut with an arch perpendicular to the Circle of any of the Hemispheres. Say

As the Radius
is to the Sine of the compl. of the altitude
of Med. celi:

so is the tangent of the circle of any Hemisphere with the Meridian,
to the tang: of the compl: of the part of
that angle, which is next the Meridian

Then subtract that part found out of the whole Angle
for the remaining or latter part.

38. To find the Distance of the cusps of
any house, from Med: celi. Say

As the Sine of the compl. of the later part of
the angle of the Ecliptic with the Meridian,
is to the Sine of the compl: of the former
part of that angle:

So is the tang. of the altitude of Med: celi,
to the tang: of the distance of the cusps of
that House sought for.

39. To find the *Altitude of the Pole above any of the circles of the Howses;*

First find out the Angle which the circle of the Howse proposed maketh with the Meridian, by Sect: 23: And then say.

As the *Radius*

is to the Sine of the *angle of the circle of the Howse with the Meridian;*

So is the Sine of the *height of the Pole above the Horizon of the place,*
to the Sine of the *height of the Pole above that circle of position.*

40. The *longitude, and latitude of any fixed Starre* being given, to find out the *Right ascension, and Declination thereof.*

The angle which the Circle of the Sunnes longitude maketh with the Meridian, at the Pole of the Ecliptic, I call the *Angle of longitude.*

And the angle which the Circle of the Sunnes Right ascension, maketh with the Meridian at the Pole of the world, I call the *Angle of right Ascension.* The condition and quantitie of which two angles, is thus found out.

In Starres of the Northerne latitude

If the longitude be in the I quadrant of the Ecliptic: subduct it out of 90: the remains will bee the *angle of longitude*, acute. And the Angle of Right ascension, being found, must be added vnto 270.

If the longitude bee in the II quadrant: subduct 90 out of it: the remains will bee the *angle of longitude*, acute. And the Angle of right ascension being found must be taken out of 270.

If the longitude bee in the III quadrant: subduct 90 out of it, the remains will be the *angle of longitude*, obtuse. And the Angle of right ascension being found, must be taken out of 270.

If the longitude bee in the IIII quadrant, subduct it out of 90+360: the remains will bee the *angle of longitude*, obtuse. And the Angle of right ascension being found, must be added vnto 270.

In Starres of the Southerne latitude

If the longitude be in the I quadrant, subduct 270 out of it + 360: the remains will be the *angle of longitude*, obtuse. And the Angle of right ascension being found, must be taken out of 90.

If the longitude bee in the II *Quadrant*, subduct it out of 270: the remains will bee the *angle of longitude*, obtuse. And the Angle of right ascension being found, must bee added to 90.

If the longitude be in the *III* quadrant: subduct it out of 270; the remaines will be the *angle of longitude*, acute. And the Angle of right ascension being found, must bee added to 90.

If the longitude be in the *IIII* quadrant: Subduct 270 out of it: the remaines will be the *angle of longitude*, acute. And the Angle of right ascension being found, must bee subducted out of 90 = 60.

Then try,

As the *Radiuse*, or totall Sine,
is to the Sine of the complement, or excesse of
the *angle of longitude*:

So is the tang. of the coropl. of the *latitude*,
to the tang. of the *first base*.

If the angle of longitude bee obtuse; vnto the *first base* found, adde the greatest declination *deg. 23*: and the summe shall be the *second base*; and the angle of right ascension shall be acute.

But if the angle of longitude be acute; out of the *first base* subduct the greatest declination: and the remaines shall be the *second base*. And the angle of right ascension shall be obtuse.

Or else out of the greatest declination of the Sun, subduct the *first base*; and the remaines shall be the *second base*: and the angle of right ascension shall be acute.

Say againe;

As the Sine of the *second base*,
is to the Sine of the *first base*:

So is the tang. of the *angle of longitude*,
to the tang. of the *angle of right ascension*.

VVhence

VVhence by adding or subducting as was before delineated in the conditions of those angles, shall be given the Right ascension of that Starre sought for.

Lastly say

As the tang. of the *second base*,
is to the *Radius* :

So is the Sine of the compl. or excess of the
angle of *right ascension*
to the tang. of *Declination*.

VVhere note that if the *second base* exceede 90 degr: the declination found, shall not be of the same kinde, that the latitude is, but in the contrary Hemisphere.

2	12	22	22	11
1	14	40	21	20
1	16	44	20	29
2	18	48	19	38
4	20	52	18	47
1	22	56	17	56
2	24	60	16	05
2	26	64	15	14
2	28	68	14	23
2	30	72	13	32
2	32	76	12	41
2	34	80	11	50
2	36	84	10	59
2	38	88	9	08
2	40	92	8	17
2	42	96	7	26
2	44	100	6	35
2	46	104	5	44
2	48	108	4	53
2	50	112	3	02
2	52	116	2	11
2	54	120	1	20
2	56	124	0	29
2	58	128	0	38
2	60	132	0	47
2	62	136	0	56
2	64	140	0	05
2	66	144	0	14
2	68	148	0	23
2	70	152	0	32
2	72	156	0	41
2	74	160	0	50
2	76	164	0	59
2	78	168	0	08
2	80	172	0	17
2	82	176	0	26
2	84	180	0	35
2	86	184	0	44
2	88	188	0	53
2	90	192	0	02
2	92	196	0	11
2	94	200	0	20
2	96	204	0	29
2	98	208	0	38
2	100	212	0	47
2	102	216	0	56
2	104	220	0	05
2	106	224	0	14
2	108	228	0	23
2	110	232	0	32
2	112	236	0	41
2	114	240	0	50
2	116	244	0	59
2	118	248	0	08
2	120	252	0	17
2	122	256	0	26
2	124	260	0	35
2	126	264	0	44
2	128	268	0	53
2	130	272	0	02
2	132	276	0	11
2	134	280	0	20
2	136	284	0	29
2	138	288	0	38
2	140	292	0	47
2	142	296	0	56
2	144	300	0	05
2	146	304	0	14
2	148	308	0	23
2	150	312	0	32
2	152	316	0	41
2	154	320	0	50
2	156	324	0	59
2	158	328	0	08
2	160	332	0	17
2	162	336	0	26
2	164	340	0	35
2	166	344	0	44
2	168	348	0	53
2	170	352	0	02
2	172	356	0	11
2	174	360	0	20
2	176	364	0	29
2	178	368	0	38
2	180	372	0	47
2	182	376	0	56
2	184	380	0	05
2	186	384	0	14
2	188	388	0	23
2	190	392	0	32
2	192	396	0	41
2	194	400	0	50
2	196	404	0	59
2	198	408	0	08
2	200	412	0	17
2	202	416	0	26
2	204	420	0	35
2	206	424	0	44
2	208	428	0	53
2	210	432	0	02
2	212	436	0	11
2	214	440	0	20
2	216	444	0	29
2	218	448	0	38
2	220	452	0	47
2	222	456	0	56
2	224	460	0	05
2	226	464	0	14
2	228	468	0	23
2	230	472	0	32
2	232	476	0	41
2	234	480	0	50
2	236	484	0	59
2	238	488	0	08
2	240	492	0	17
2	242	496	0	26
2	244	500	0	35
2	246	504	0	44
2	248	508	0	53
2	250	512	0	02
2	252	516	0	11
2	254	520	0	20
2	256	524	0	29
2	258	528	0	38
2	260	532	0	47
2	262	536	0	56
2	264	540	0	05
2	266	544	0	14
2	268	548	0	23
2	270	552	0	32
2	272	556	0	41
2	274	560	0	50
2	276	564	0	59
2	278	568	0	08
2	280	572	0	17
2	282	576	0	26
2	284	580	0	35
2	286	584	0	44
2	288	588	0	53
2	290	592	0	02
2	292	596	0	11
2	294	600	0	20
2	296	604	0	29
2	298	608	0	38
2	300	612	0	47
2	302	616	0	56
2	304	620	0	05
2	306	624	0	14
2	308	628	0	23
2	310	632	0	32
2	312	636	0	41
2	314	640	0	50
2	316	644	0	59
2	318	648	0	08
2	320	652	0	17
2	322	656	0	26
2	324	660	0	35
2	326	664	0	44
2	328	668	0	53
2	330	672	0	02
2	332	676	0	11
2	334	680	0	20
2	336	684	0	29
2	338	688	0	38
2	340	692	0	47
2	342	696	0	56
2	344	700	0	05
2	346	704	0	14
2	348	708	0	23
2	350	712	0	32
2	352	716	0	41
2	354	720	0	50
2	356	724	0	59
2	358	728	0	08
2	360	732	0	17

M 3

A Table

A Table of the Right Ascensions, and
Declinations, of 40 of the cheifest fixed
Starres, Calculated for the yeare of
our Lord. 1650.

Names of Starres	Right Ascension	Declination	mag.
The Polar Starre	7 ^h 47 ^m 30 ^s	27 [°] N	2
Andromedaes girdle	13 32 33	48 N	3
the former horne of Υ	23 38 17	37 N	4
the bright starre in the head of Υ	26 56 21	48 N	3
the iawe of the Whale	41 03 2	43 N	2
Medusaes head	41 27 39	35 N	3
the eye of the Bull	64 00 15	46 N	1
the Goat starre	72 44 45	35 N	1
the former shoulder of Orion	76 38 4	59 N	2
the latter shoulder of Orion	84 07 7	18 N	3
the great dogge starre	97 27 16	13 S	1
the higher head of π	108 01 32	35 N	2
the lesser Dogge starre	110 17 6	06 N	2
the lower head of π	111 00 28	49 N	2
the Cribb, or Manger	125 4 20	53 N	Neb.
the heart of Hydra	137 39 7	10 S	2
the heart of the Lion	147 27 13	39 N	1
In the loynes of the Lion	163 54 22	26 N	2
In the tayle of the Lion	172 49 16	32 N	1
In the girdle of Virgo	189 32 5	20 N	3

Names of Starres	R. Ascen.	Declination	mag.
Aliot	189° 36 57 53	N	2
Vindemiatrix	191 15 15 51	N	3
Spica Virginis	196 44 9 17	S	1
Arcturus	209 56 21 34	N	1
the Southerne ballance	217 56 14 32	S	2
the Northerne ballance	224 31 8 2	S	2
the bright star in the serpens neck	231 49 7 35	N	3
the heart of Scorpius	242 4 25 34	S	1
the head of Hercules	254 40 14 51	N	3
the head of Ophiucus	259 41 12 52	N	3
bright starre in the Harp	276 17 38 30	N	1
bright starre in the Vultur	293 27 8 1	N	2
upper horne of ♊	299 30 13 32	S	3
the left hand of ♊	307 10 10 43	S	4
the left shoulder of ♊	318 18 7 2	S	3
the mouth of Pegasus	331 49 8 18	N	3
the right shoulder of ♊	326 59 1 58	S	3
Fomalant	339 29 31 23	S	1
In the upper wing of Pegasus	341 53 13 21	N	2
In the tip of the wing of Pegasus	358 52 13 15	N	2

41. The *longitude*, and *latitude* of any
two *Starres* being giuen, to finde their
distances.

If the *Starres* haue both the same *longitude*, differing
only in *latitude*; the difference of *latitude*, is the distance
of the *starres*.

And if they differ onely in *longitude* hauing the same
latitude; Say

As the *Radius*,
is to the Sine of *halfe the difference of longitude*;
So is the Sine of the compl. of the *latitude* giuen,
to the Sine of *halfe the distance of the Starres*.

But if they differ both in *longitude*, and *latitude*, whe-
ther the *latitudes* be both of the same kinde, or one Nor-
therne, and the other Southerne. Take the difference of
both the *starres*, from the pole of the *Ecliptick*, toward
which the *starre* hauing the greater *latitude* is. And say,

As the tang. of the compl. of the *lesser distance*
from the pole,
is to the *Radius*;
So is the Sine of the compl. or *excesss of the dis-
tance of longitudes*,
to the tang. of the *first base*.

Take this *first base* out of the greater distance from the
pole, and the remaines shal be the *second base*. Then say,

As the Sine of the compl. of the *first base*,
is to the Sine of the compl. of the *second base*;
So is the Sine of the compl. of the *lesser distance*
from the pole,
to the Sine of the compl. of the *distance of the*
two Starres.

If any man will take paines to calculate (by this last Rule) the distances of some noted stars of the first, second, and third magnitudes, round about the heavens, which are not above 5, or 6 degrees, at the most, one from the other: and shall keepe them written in his booke: they may serue as a Rule, or Instrument, whereby he may reasonably estimate with his eye, the distance of any Planer, or Comet, or other apparition from a known fixed starre, not very farre remote: by comparing the distance which hee would know with some of those known distances which he shall find, either to be equal, or else to haue some proportion thereto.

42. *The longitude, and latitude of any two Cities being giuen, to find their distance.*

The manner of the operation is the very same with the former, vnto which therefore I referre the Reader: onely will note, that in the heavens, the longitude and latitude is taken in respect of the Ecliptic, which being the way of the Sunne, all the stars in their proper motion, haue reference vnto it, as vnto their measure and rule. But in the Earth the principall Circle is the Equinoctiall, diuiding it into the Northerne, and Southerne hemispheres. And therefore in the earth, the longitude, and latitude is reckoned by the Equinoctiall.

The distance of two places vpon the Earth, being found in degrees, may bee conuerted into English miles, by taking 60 miles for euery degree, and one mile for euery minute.

43. *To find at What hower a fixed starre commeth into the Meridian any day.*

Seeke the Right ascension of the Sunne, for that day, by Sect. *ii*; and subduct it out of the Right ascension
N of

of the Starre. And reduct the degrees remaining into howers, by Sect: 1. The same shall shew how long time from the Noone before, the same starre shall come into the Meridian.

Wherefore if at any time of the night, a Starr whose Right ascension is knowne, be in the Meridian, the hower of the night is easily found.

44. The height of any knowne Starre above the Horizon, being by any means given, to find the hower of the night.

First seeke out the hower of that starre comming into the Meridian the same day, by Sect. 43. Againe seeke out the horary distance of that starre from the Meridian, according to Sect: 19. And then if the starre bee on the East side, not yet come to the Meridian, take the difference of those two numbers; but if the starre bee past the Meridian, take the Summe of them, for the houre of the night.

45. The height of the Pole being given to find the comming of any fixed Starre, in the due East, or West. Say

As the Radius

is to the tang: of the starres declination:

So is the tang: of the compl: of the Pole, to the Sine of the compl. of the Starres horary distance from the Meridian.

46. The height of the Pole being given, to find the Altitude of any fixed star above the Horizon, being due East or West. Say,

As

As the Sine of the height of the Pole,
is to the Radius :
so is the Sine of the Stars declination,
to the Sine of the Altitude, at due East or West.

47. By the Altitudes of any two known
fixed stars taken when they are both in the
same Azimuth, to find the height of the
Pole.

First say,

As the Sine of the difference of the stars altitudes,
is to the Sine of the difference of their Right ascensions :
so is the Sine of the nearer stars distance from the
apparent Pole,
to the Sine of an angle to be kept.

Again compare the furthest stars distance from the
Pole with the distance from the Zenith, and say

As the Radius
is to the Sine of the compl. of the angle kept :
so is the tang : of the lesser of the compared arches,
to the tang : of the first base.

Subduct the first base out of the greater of the two
compared arches ; and the remains shall bee the second
base.

Then lastly say,

As the Sine of the complement of the first base,
is to the Sine of the compl. of the second base :
so is the Sine of the compl. of the lesser of
the two compared arches,
to the Sine of the height of the Pole.

48. To find out the *horizontal Parallax of the Moone.*

First the distance of the Moone from the Center of the earth must be knowne in Semidiameters of the earth: which vnto them that are acquainted with the Theoric of the Planets, is not very difficult. And whereof peradventure, I may hereafter teach the practise, by most easie and exact instruments, which I haue long since framed.

Say,

As the *distance of the Moone, from the center of the earth,*
is to the *Semidiameter of the earth:*
So is the *Radius,*
to the Sine of the *Moones horizontal parallax in that distance.*

49. The *horizontal Parallax of the Moone* being given, to find her *Parallax in any apparent altitude.*

As the *Radius,*
is to the Sine of the *altitude of the Moone:*
so is the Sine of the *horizontal Parallax,*
to the Sine of the *Parallax in that altitude.*

50. The *place of the Moone in the Ecliptic* hauing little or no latitude (as in the Eclipse of the Sunne) together with her *Parallax of altitude* being giuen, to find the *Parallaxes of her longitude, and latitude.*

If the Moone bee in the 90th degree of the Ecliptic: shee hath no Parallax of longitude, and the Parallax of latitude, is the very Parallax in that altitude.

But

But if the Moone be not in the 90th deg. say,

As the *Radius*

is to the tang. of the angle of the *Ecliptick* with
the horizon :

So is the Sine of the compl. of the distance of the
Moone, from the *Ascendent*, or *descendent de-*
grees of the Ecliptick,

to the tang. of the compl. of the angle of the
Ecliptick with the *Azimuth of the Moone*.

Again say,

As the *Radius*

is to the Sine of that angle :

So is the *Parallax of the Moones altitude*,
to the *Parallax of her latitude*.

Lastly say,

As the *Radius*

is to the Sine of the compl. of that former angle :

So is the *Parallax of the Moones altitude*,
to the *Parallax of her longitude*.

which is to bee added to the true motion of the Moone, if
she be on the East part of the 90th degree of the *Ecliptick*;
or to be subducted out of it, if she be on the West part.

Many other *Astronomickall* and *Geographickall* problems
might be added. But because it is impossible to set downe
all, which may be of vse, at some time or other : I haue
in the next Chapter deliuered briefly the doctrine of tri-
angles fitted vnto practise : with all the senerall cases be-
longing thereto.

CHAP. XIII.

of Trigonometria, or the manner of calculating both Plaine, and Sphæricall triangles. And first concerning certaine generall notions, and rules necessary thereto.



In every triangle both Plaine, and Sphærical, the greater side subtendeth the greater angle. And the greater angle hath the greater side opposite vnto it. Also the greater angle lyeth to the lesser side, and the greater side hath the lesser angle lying vnto it.

In every plaine triangle, any two angles being giuen, the third is also giuen: and one of the angles being giuen, the summe of both the other two is giuen. For all the three angles together, are equall to two right angles, that is to 180 degrees.

In a plaine rectangle triangle, one of the acute angles is the complement of the other. Where note that when the complement is named without any other addition, it is meant of the arch, which is wanting of a quadrant of that circle, or 90 degrees. In like manner the excess is meant of the arch, which is aboue a quadrant. But when it is said the complement to a semicircle, it is vnderstood of so many degrees as will make vp 180.

But in a Sphæricall rectangle triangle, one of the oblique angles is alwayes greater then the compl of the other.

If two arches together make vp a Semicircle, the excess of the greater arch, is equall to the complement of the lesser.

The same Right Sine, and the same Tangent, and Secant, doth belong both to the arch it selfe, and also to the complement of it to a Semicircle. But their versed Sines differ: For the versed Sine of an arch lesse then a quadrant, is equall to the difference of the *Radius*, and the Sine of the complement of that arch: and the versed Sine of an arch greater then a quadrant, is equall to the summe of them. And the versed Sine is thus found out, As the *Radius* is to the Sine of halfe the arch; so is the Sine of that halfe arch, to halfe the versed Sine of the whole arch.

In a right angled triangle both Plaine, and Sphaericall, one of the sides containing the right angles, is called the Base, and the other the Cathetus: and the side subtending the right angle is the Hypotenusa. And know that every rectangled triangle, is most fitly noted with the letters *ABC*; so that *BA* may be the Base, and *CA* the Cathetus, and *BC* the Hypotenusa: and *B* the angle at the base, and *C* the angle at the Cathetus, and *A* the right angle. Likewise every oblique-angled triangle with the letters *BCD*; so that out of the angle *C* a perpendicular *CA*, being let downe, it may in the base *BD* distinguish the two cases *BA* and *DA*, which are the bases of the two particular triangles into which it is cut. And in noting the triangles with letters, observe diligently, that if any angle be given together with one of the sides including it, the same angle be noted with *B*; and the side with *BC*.

If both the angles at the Base *BD* be acute, the perpendicular *CA* shall fall within the triangle: And $BD = BA + DA$ that is *BD* is equall to the summe of *BA* and *DA*. And if the angle *B* be obtuse, the perpendicular *CA* shall fall without the triangle, beyond the obtuse angle *B*: And $BD = DA - BA$, that is *BD* is equall to the excessse of *DA* above *BA* or if the angle *B*. be obtuse, the perpendicular *CA*: shall fall without the triangle beyond the obtuse

angle D : And $BD = BA - DA$, that is BD is equall to the excess of BA above DA . The lesser case being still taken from the greater angle.

And note that this signe $+$, or p (that is *plus*) sheweth that the magnitude before which it is set, is affirmed and positive in nature; and therefore to bee added. And that this signe $-$ or m (that is *minus*) sheweth the magnitude before which it is placed, is denied, and priuative in nature; and therefore to be subtracted, as you may see in those former examples.

Again, some magnitudes are taken severally and apart; as $s BA$, that is the Sine of the Base; $s co BC$, that is the Sine of the complement of the Hypotenusa; $s B$, that is the tangent of the angle ABC at the base; $s co C$, or $s co ABC$, the tangent of the complement of the angle at the Cathetus: So also $s q Z$, that is the quadrat side of the plaine Z . And some magnitudes are taken vniuersally, and then they are included in prick: as $s \frac{DC+BD-BC}{2}$;

that is the Sine of halfe the arch, which is composed of the summe of the two arches DC , and BD , abating thereout the arch BC . So also $s q : Z \cdot X$: that is the quadrat side of the two plaines Z and X put together: also $s q : Q$ in R : Or $s q : Q \cdot R$: that is the quadrat side of a rectangular plaine, the two sides whereof are the lines Q and R , or some fourth proportionall already found, and the *Radius*, or Semidiameter, which is the totall Sine. For by the signe in, or \cdot , I vse to expresse multiplication.

When any triangle is giuen to be resolued by Trigonometrie, note the parts thereof (either sides or angles) which are giuen and knowne, with a little line drawne crosse each such part: and note the vknowne part which is sought for with a little circle.

And

And if a triangle Sphæricall (bee it right angled or oblique-angled) proposed hath two sides each of them severally greater then a quadrant: you shall in resolving thereof, keepe the least side with the least angle opposed to it: and for the two other both sides, and angles, take the complements of them to a semicircle.

Lastly, if a triangle with all the three angles given, be required to be converted into a triangle having the three sides given. You shall for the greatest angle of the triangle proposed, & for the greatest side subtending it, take the complements to a semicircle; keeping the other two lesser angles, with their subtendent sides as they are.



THE

THE CALCULATION OF PLAINE

right-angled-triangles.

I.



$$BC : BA :: R : \sin B (\text{or } C) :$$

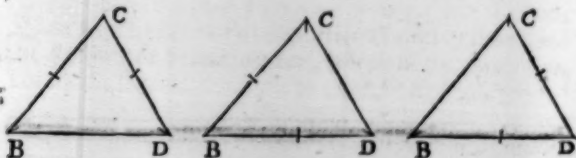
II.



$$BA : CA :: R : \sin B (\text{or } C) :$$

Of plaine Oblique-angled triangles.

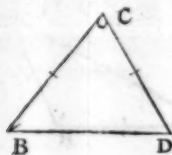
III.



$$\sin B : DC :: \sin D : BC :: \sin C : BD .$$

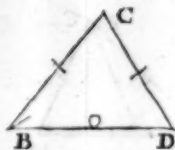
and here it is necessary to be knowne, whether the angle sought for be greater, or less then a right angle, or 90 deg.

Full



III.

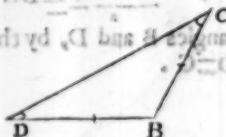
First seeke the angle D, by the III; then both the angles B and D being subducted out of 180, you shall haue
 $180 - B - D = C$.



V.

First seeke the angle D, by the I; then both the angles B and D being subducted out of 180, Say

$$B : DC :: 90 : 180 - B - D \therefore BD$$



VII

Let the side BD be greater then the side BC:

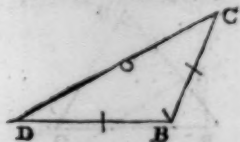
$$\text{First, } BD : BC :: BD - BC :: \frac{180 - B}{2} + Q$$

then for the other two angles:

$$\frac{180 - B}{2} + Q \text{ the greater} \quad \frac{180 - B}{2} - Q \text{ the lesser.}$$

Let

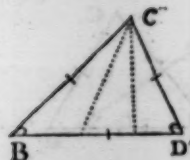
VII.



Let the side BD be greater then the side BC :
 First, the angles C and D are to be sought, by the VI.
 and the side DC, by the III.

then

VIII.



Take the greatest side BD for the base : and let the side
 BC, be greater then the side DC : First say,

$BD : BC + DC :: BC - DC : Q$ (viz $BD - DA$) :

then $\frac{BD+Q}{2} = BA$. $\frac{BD-Q}{2} = DA$.

Nextly seeke the angles B and D, by the III :

Lastly $180 - B - D = C$.

THE

THE CALCVLATION OF
Sphæricall right-angled, and quadrantall
triangles.



I.

$$R : \sin B :: \sin BC : \sin CA :$$



II.

$$R : \sin B :: \sin BA : \sin CA :$$



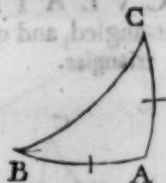
III.

$$R : \sin BA :: \sin CA : \sin BC :$$

Q 3

R : sin

III.



$$R : \sin BA :: \sin B : \sin CA.$$

$$R : \sec B :: \sin CA : \sin BA.$$

$$R : \sin BA :: \sec CA : \sec B.$$



$$R : \sec B :: \sin BC : \sin BA.$$

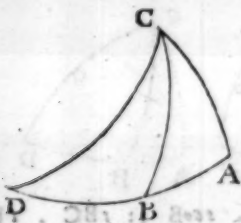
$$R : \sin BC :: \sin BA : \sec B.$$

$$R : \sec B :: \sec BA : \sec BC.$$



$$R : \sec BC :: \sin B : \sin C.$$

If



VII.

If a triangle BCD be quadrantall, having one side BC equal to a quadrant; upon the pole D describe an arch of a great circle CA, cutting the side DB extended in A: and so making a right-angled triangle ABC without the other. This outward right-angled triangle shall be resolved in stead of the quadrantall proposed.

Of Spharicall Oblique-angled triangles.



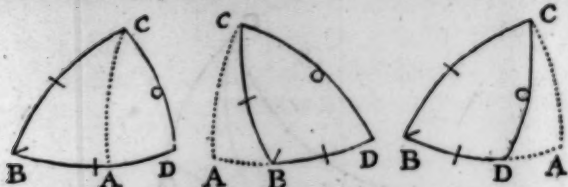
VIII.

$\angle B :: \angle DC :: \angle D :: \angle BC :: \angle C :: \angle BD ::$

and in these it is necessary to bee knowne whether the terme sought for be greater then a quadrant, or not. The same also is to be knowne in the tenne rules next following, if the sides BC and DC are both given.

First,

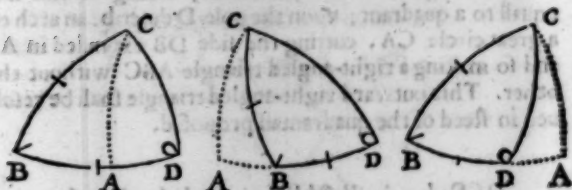
IX.



First, $R : \cos B :: \sin BC : \sin BA :$

then, $\sin BA : \sin DA :: \sin BC : \sin DC :$

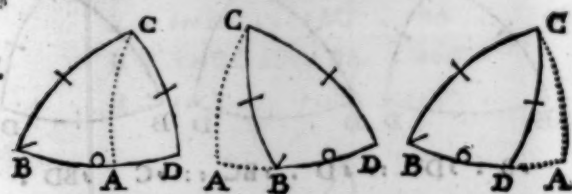
X.



First, $R : \cos B :: \sin BC : \sin BA :$

then $\sin DA : \sin BA :: \sin B : \sin D .$

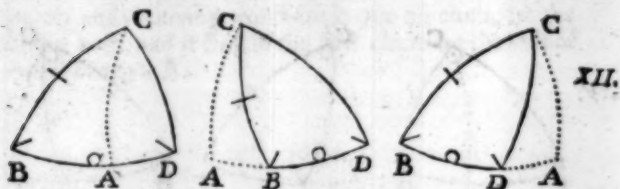
XI.



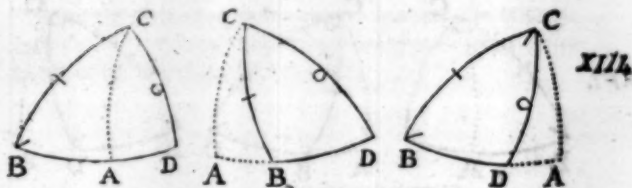
First, $R : \cos B :: \sin BC : \sin BA :$

then, $\sin BC : \sin DC :: \sin BA : \sin DA :$

First,



First, $R . \sec B :: \sec BC . \sec BA :$
 then, $\sec D . \sec B :: \sec EA . \sec DA :$

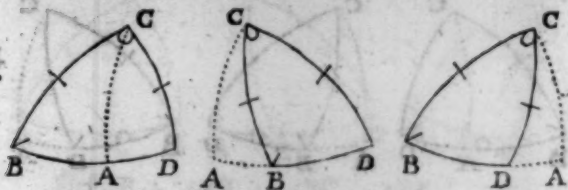


First, $R . \sec BC :: \sec B . \sec BCA :$
 then, $\sec DCA . \sec BCA :: \sec BC . \sec DC :$



First, $R . \sec BC :: \sec B . \sec BCA :$
 then, $\sec BCA . \sec DCA :: \sec B . \sec D :$

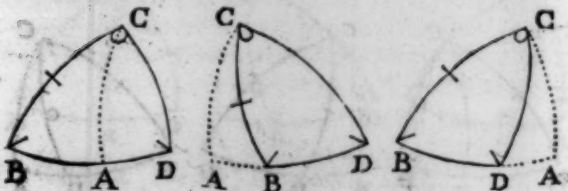
xv.



First, $R . \sec BC :: \sin B . \sec BCA .$

then, $\sin DC . \sin BC :: \sec BCA . \sec DCA .$

xvi.



First, $R . \sec BC :: \sin B . \sec BCA .$

then, $\sec B . \sec D :: \sin BCA . \sin DCA .$

xvii.

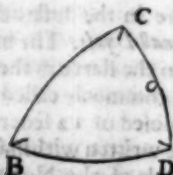


First, $R . \sin BD :: \sin BC . \sin QI .$

then, $\sin QI :: \frac{DC + BD - BC}{2} :: \frac{DC + BC - BD}{2} . QII .$

See what QII cutteth in the fifth circle, which is of equal divisions: and thereto add the Radius, by setting 1 before

1 before that number. Divide the whole into two equal parts: and reckoning one halfe in that fifth circle, set the Index to it, and it shall in the first circle cut the Sine of halfe the angle B.



XVIII.

If all the three angles be given: conuent the triangle into another haning all the three sides giuen: and resolue the same for the triangle proposed.

THE
P a

to find out the hour of the night
by Altitude

Set the day of the month in the small circle of the
Machinall: and apply the Index thereon: make what
hours it comesh, in the hour circle. Remember this
house

CHAP. XIII.

of the Nocturnall Dials.



Here are in the Instrument, two severall *Nocturnall Dyals*. The innermost of them is fitted to the starre in the rump of the great Beare, commonly called *Aliot*. The other is composed of 12 severall starres: whose names you shall finde written within, neere to the center.

The outermost circle of the Nocturnall Dyall is divided into twise 12 houres: each houre being subdivided into quarters, and are noted with figures belonging to the houres, as may be seene in the Instrument.

The middlemost circle of the Nocturnall is divided into 12 moneths, having their names written: each moneth being distinguished into tenth dayes with longer lines; and into fift dayes with shorter lines. And if the Instrument be large enough, each day of the monethes throughout the year, may be noted with pricks.

In the innermost circle are the dimissions and names of 12 fixed starres: which are these.

The bright star in the head of γ .
the Bulls eye
the latter shoulder of Orion
the little dogge
the heart of the Lyon
the tayle of the Lyon

Spica Virginis
the North ballance
the head of Ophiuchus
the heart of the Vultur
the mouth of Pegasus
the tip of the wing of Pegasus.

*To finde out the houre of the night
by Aliot.*

Seeke the day of the moneth in the annuall circle of the Nocturnall: and apply the Index thereto: marke what houre it cutteth, in the houre circle. Remember this
houre

houre for all that day : then at night when you would finde out the houre, hold vp your Instrumēt by the handle, and moue it vp and downe, till you see the pole starre through the middle hole, and the starre called *Aliot* by the limbe : Set the Index or labell to *Aliot*, and marke what houre the labell cutteth, for if vnto this houre you adde the houre kept in minde for that day, the summe of both shall giue you the true houre of the night : so that you cast out 12 houres, from the said summe if it shall chance to be more.

Example. If on the 15th of Nouember you would finde the houre of the night by the starre *Aliot*. Apply the Index to the day of the moneth, and it will cut in the houre circle 8 and an halfe : then suppose the Index being set to the starre *Aliot*, as aboue taught, doth cut in the houre circle 10. these two numbers being added together, the summe wil be $18\frac{1}{2}$, out of which subduct 12 ; and the remaines 6 $\frac{1}{2}$, will be the true houre of the night.

*To finde out the houre of the night by the
Inner Nocturnall Dyall.*

To performe this it is necessary that you know the true Meridian of the place wherein you are, and can finde it out by night, which you may thus doe. Haneing a Meridian line drawne in some window, or other conuenient place (as is shewed in the Second part of this booke, *Vto 19*) stick vp therein a long needle perpendicularly, and watch till the Sunne casteth the shadow of the needle, vp-on the Meridian line. Or else in a true Sunne Dyall obserue when the shadow falls iust on 12 a clock, for then is the Sunne in the Meridian. Wherefore goe instantly into some place about your house where you may see some marke, either a chimney, or the corner of an house, or else some tree, or such like, directly betweene you and the Sunne : then haue you the true Meridian.

Or otherwise you may in a cleare night goe into some plaine place neere your house, and setting vp a straight pole perpendicularly on the ground, goe a good distance from it Southwards; and then moue vp and downe, till you see the top of your pole, directly betweene your eye, and the North polar starre: then set vp another pole perpendicularly betweene your feet, so that both your poles, and the Polar starre, may be in one right line. And then going backe againe to your first pole, looke what knowne starre is directly ouer your last pole, for that starre is in the Meridian. You may therefore instantly goe to some conuenient place, and take a marke whereby you may at all times know the Meridian as is afore taught.

When therefore at any time of the night you would know what a clocke it is, goe to that place where you stood, and looking directly ouer your marke, see if any of the 12 fixed starres, bee in the Meridian; or if none of them be therein, obserue which two of them are on either side thereof, and what part of that space is in the Meridian. Then goe into the light, and take your instrument, and set the Index to that starre, or point which you saw in the Meridian: marke what houre it cutteth, for that same houre being added to the houre, which the day of the moneth sheweth, shall giue you the true houre of the night: so that you cast out 12 houres, from the said summe, if it shall chance to be more.

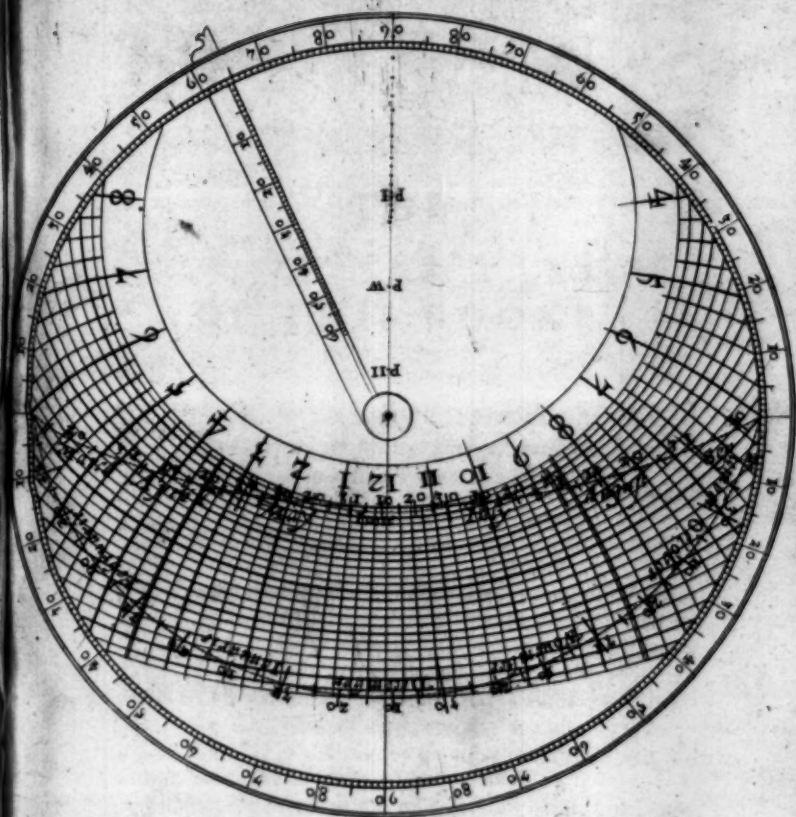
Example. Suppose the fifth of December, that the middle point of the space betweene the bright starre in the head of *Aries*, and the *Bulls eye*, bee in the Meridian. Set the Index to the middle point of the space betweene those two starres in the Instrument: and it will cut in the houre circle 2 and an halfe: then againe set the Index to the fifth of December, and in the houre circle it wil cut 7: which

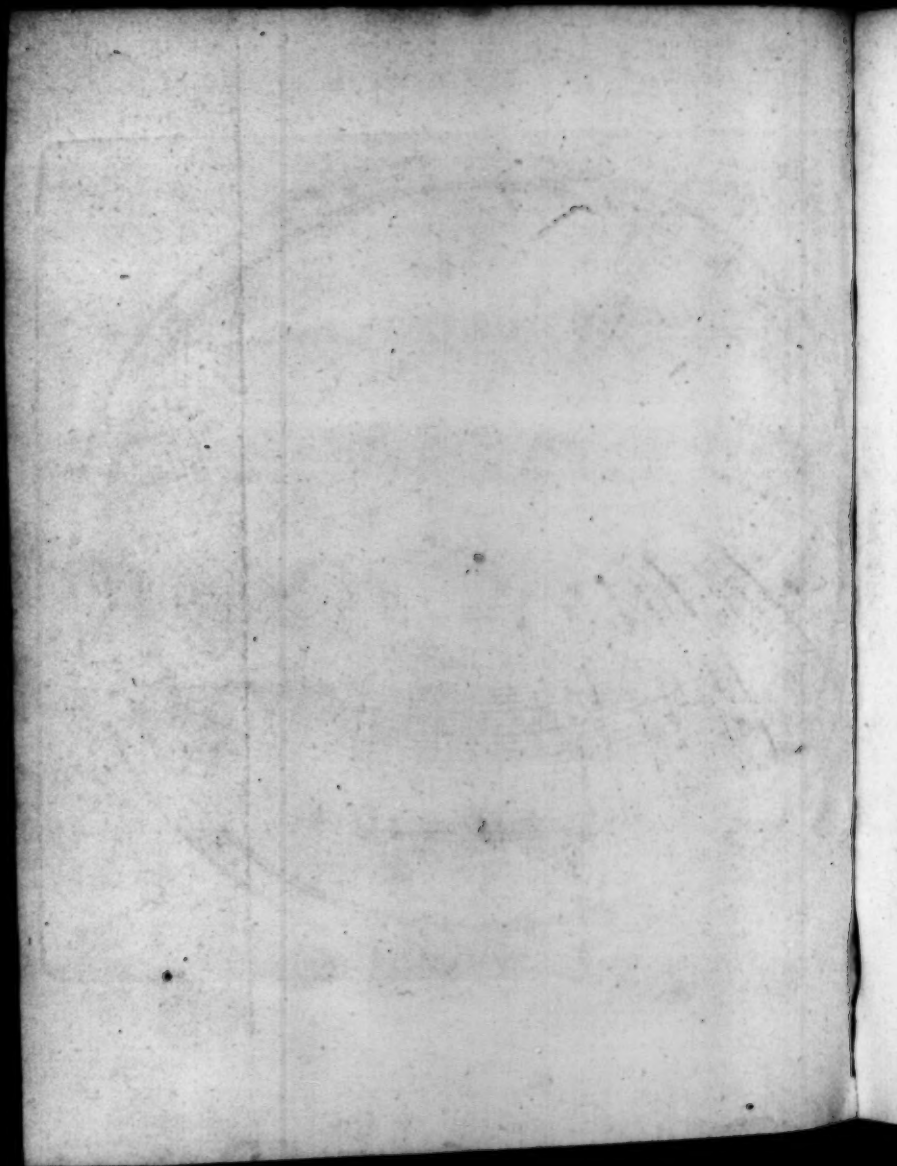
which added vnto 3 and an halfe, giueth 9 and an halfe, for the true houre of the night.

Another example. Suppose the 19th of December, that one third part of the space betweene the *Bulls eye*, and the *right shoulder of Orion*, be in the Meridian. Set the Index to one third part of that space in the Instrument, and it will cut in the houre circle 4 and halfe a quarter almost: againe, set the Index to the 19th of December, and in the houre circle it will cut 6, which being added vnto 4, and halfe a quarter almost, giueth 10 and almost halfe a quarter for the houre of the night.

[illegible]

.....







THE SECOND PART OF THIS BOOKE.

*Shewing the vse of the Second side of the
Instrument, for the working of most questio-
ns, which may be performed by the Globe:
And the declination of Dyals, vpon any
kinde of Plaine.*



Pon the *second side* of the Instrument, is delineated the *projection of the upper Hemisphere vpon the plaine of the Horizon*: The *Horizon* it selfe is vnderstood to bee the innermost circle of the limbe: and is diuided on both sides, from the points of *East*, and *West* into degrees, noted with 10, 20, 30, &c. vnto 90. And the center of the Instrument is the *Zenith*, or *Vertical point*.

Within the *Horizon*, the middle straight line, or Diameter pointing North and South, is the *Meridian*, or 12 a clock line: and the other short arching lines, on both sides of it are the *houre lines*, distinguished accordingly by their figures.

figures. These houre lines should indeede bee drawne through the whole plaine, crossing one another in the Pole of the world: but that the Instrument may be more faire, they are onely drawne short.

And because diuers excellent vses, doe require the totall delineation of the houre circles, I haue in a seuerall paper, inscribed intirely, both the houre lines, and also two other circles betweene them, containing every one five degrees. (But if the Instrument were large enough to receive them, it were best if every degree had his circle: and so every 15 circle should bee an houre line.) And of the parallels, there needes no more, but the *Equinoctial*, and both the Tropics.

For as much as there will be great vse of this paper Instrument; I haue in the 24 Vse shewed the manner of making it: so that any that is ingenuous, and ready handed may himselfe delineate one sufficient enough to serue his turne, for any eleuation.

The two arches which crosse the houre lines meeting on both sides in the points of intersection of the fixe a clocke lines with the Horizon, are the *two Semicircles of the Eclipse*, or Annuall circle of the Sunne: the vpper of which arches serueth for the *Summer halfe yeare*, and the lower for the *Winter halfe yeare*: and are therefore diuided in 365 dayes: which are also distinguished into 12 moneths with longer lines, hauing their names set downe: and into tenthes, and fiftes with shorter lines: and the rest of the dayes with pricks: as may plainly bee seene in the Instrument.

And this is for the ready finding out of the *place of the Sunne* every day: and also for shewing of the *Sunnes yearely motion*: because by this motion the Sunne goeth round about the heauens in the compasse of a yeare, making

ing the *four parts*, or *seasons* thereof. Namely the *Spring*, in that quarter of the Ecliptick which beginneth at the intersection on the West side of the Instrument, and is therefore called the *Vernal intersection*. Then the *Summer* in that quarter of the Ecliptick which beginneth with the intersection of the Meridian in the highest point next the Zenith. And after that *Autumne* in that quarter of the Ecliptick, which beginneth at the intersection on the East side of the Instrument, and is therefore called the *Autummall intersection*. And lastly, the *Winter* in that quarter of the Ecliptick, which beginneth at the intersection with the Meridian, in the lowest point next the Horizon.

But besides this yearely motion, the Sunne hath a *Diurnall* or *dayly motion*, whereby it maketh day and night with all the diuerſities, and inequalities thereof: which is expreſſed by thoſe other circles drawne croſſe the houre lines: the middlemoſt whereof being groſſer then the reſt, meeting with the Ecliptick in the points of the *Vernal*, and *Autummall intersections*, is the *Equinoctiall*: and the reſt on both ſides of it, are called the *Parallels*, or *Diurnall arches of the Sunne*: the two outermoſt whereof are the *Tropics*, becauſe in them the Sunne hath his furtheſt *digreſſion*, or *Declination* from the *Equinoctiall*, which is degrees 23½; and thence beginneth againe to returne to the *Equinoctiall*. The vpper of the two *Tropics* next the center (in this our Northerne Hemisphere) is the *Tropick of Cancer*: and the Sunne being in it, is higheſt into the North, making the longeſt day of Summer. And the lower next the Horizon, is called the *Tropick of Capricorne*; and the Sunne being in it, is loweſt into the South, making the ſhorteſt day of Winter.

Betweene the two Tropics, and the *Equinoctiall* infinite ſuch *parallel circles* are vnderſtoode to be contained;

contained: for the Sunne is what point focuer of the Ecliptick it is caried, describeth by his lation, a circle parallel to the *Æquinoctial*. Yet those parallels which are in the Instrument, though drawne but to euery second degree of Declination, may be sufficient to direct the eye, in imagining and tracing out, through euery day of the whole yeare in the Ecliptick, a proper circle, which may be the *Diurnall arch of the Sunne* for that day. For vpon the right estimation of that imaginary parallel, doth the manifold vse of this Instrument especially rely: because the true place of the Sunne, all that day, is in some part, or point of that circle. Wherefore for the better conceiuing, and bearing in minde thereof, euery fift parallel, is herein made a little grosser then the rest.

I Vse. And thus by the eye, and view only, to behold and comprehend the course of the Sunne, both for his Annuall and Diurnall motion, may be the first vse of this Instrument.

II Vse. To take the height of the Sun above the Horizon.

Set vp the pinne, (which is therefore made fit for the hole at the center) perpendicular in the center: and put the Indices on both sides, downe vpon the Meridian, that they with their waight, may not sway the Instrument any way as it hangeth: then with a threed put into the hole aboue in the handle, hang it perpendicularly, bearing the edge toward the Sunne, that the pinne may cast a shadow, vpon the degrees in the limbe: for that degree which the shadow of the pinne cutteth in the limbe, is the height of the Sunne above the Horizon, at that present.

III Vse.

III. Vſe. To find the Declination
of the Sunne every day.

Looke the day of the moneth propoſed in the Ecliptic, and marke how many degrees the prick ſhewing that day, is diſtant from the Equinoctiall, either on the Summer, or Winter ſide, viz North, or South.

Example. I. What will the Declination of the Sunne be, vpon the 11th day of *Auguſt*? Looke the 11th day of *Auguſt*: and you ſhall find it in the fixt Circle about the Equinoctiall: now becauſe each Parallel, ſtandeth (as hath beene ſaid before) for 2 degrees, the Sunne ſhall that day decline North-wards 12 degrees.

Example. II. What Declination hath the Sunne, vpon the 24th day of *March*? Looke the 24th day of *March*, and you ſhall find it, betweene the ſecond, and third Northerne parallels, as it were an halfe and one fiſt part more of that diſtance from the ſecond: reckon therefore 4 degrees for the two Circles, and one degree for the halfe ſpace: ſo ſhall the Sunnes declination bee 5 degrees, and about one fiſt part of a degree Northward; that ſame day.

Example. III. What Declination hath the Sunne vpon the 13th day of *November*? Looke the 13th day of *November*, and you ſhall find it below the Equinoctiall, tenne parallels and about one quarter, which is 20 degrees, and an halfe South-wards. So much is the Declination. And according to theſe examples iudge of all the reſt.

IIII. Vſe. To find the Right aſ-
cenſion of the Sunne every day.

Imagine an hower line through the day of the moneth giuen, and marke in what point it will crosse the

Æquinoctiall : then lay a Ruler, or a ſtreight Scroule of paper, to the Pole of the world (noted in the Inſtrument with *PW*) and that ſame point. For the Ruler ſhall in the innermoſt Circle of the limbe, of the South ſide, cut the Right aſcenſion of the Sunne for that day, to be reckoned from the Weſt, to the point of interſection, for the firſt, or vpper Semicircle of the Ecliptic : or from the Eaſt together with 180, for the ſecond, or lower Semicircle of the Ecliptic.

V. Vſe. *To find the longitude of the Sunne, or in what degree of the Signe he is every day.*

The Pole of the firſt Semicircle of the Ecliptic is noted *P.I.* and the Pole of the ſecond Semicircle is noted *P.II.* Lay a Ruler, or a ſtreight Scroule of paper, to the day of the moneth, and the proper Pole of the Semicircle of the Ecliptic, in which it is : for the Ruler ſhall in the innermoſt Circle of the limbe, on the South ſide, cut the degree of the Sunnes place in the Ecliptic, reckoning it in the ſame manner as you did in finding the Sunnes Right aſcenſion : and the Arch thus found is called the longitude of the Sunne. which may bee expanded into ſignes, by reckoning on the limbe, from the Weſt to South γ, δ, π , and from South to Eaſt σ, η, θ : then backe againe from Eaſt to South ϵ, μ, τ ; and laſtly from South to Weſt ψ, α, κ , allowing 30 degrees, for each of thoſe twelve ſignes.

VI. Vſe. *To find the Diurnall Arch, or Circle of the Sunnes courſe every day.*

The Sunne every day by his motion (as hath bene ſaid) deſcribeth a Circle parallell to the *Æquinoctiall*, which is either one of the Circles in the Inſtrument, or ſome

some-where betweene two of them. First then seeke the day of the moneth; and if it fall vpon one of those Parallels, that is the Circle of the Sunnes course that same day: But if it fall betweene any two of those Parallel, imagine in your minde, and estimate with your eye, another Parallel through that point, betweene those two Parallels, keeping still the same distance from each of them.

As in the first of the three former Examples, The circle of the Sunnes course, vpon the 11th day of *August*, shall be the very sixt Parallel about the *Æquinoctiall* towards the Center.

In Example II. The Circle of the Sunnes course vpon the 24th day of *March*, shall bee an imaginary Circle betweene the second, and third Parallels, still keeping an halfe of that space, and one fift part more of the rest from the second.

In Example III. The Circle of the Sunnes course vpon the 13th day of *November*, shall be an imaginary Circle, betweene the tenth, and eleuenth Parallels, below the *Æquinoctiall*, still keeping one quarter of that space from the tenth.

VII. Vse. *To find the Rising and Setting of the Sunne every day.*

Seeke out (as was last shewed) the imaginary Circle or Parallel of the Sunnes course, for that day, and marke the point where it meeteth with the Horizon, both on the East and West sides thereof, for that is the very point of the Sunnes rising, and setting that same day, and the hower lines which are on both sides of it, by proportioning the distance reasonably, according to 15 minuts, for the quarter of the hower, will shew the hower of the Sunnes rising, on the East side, and the Sunnes setting on the West side.

VIII. Use. *To know the reason, and manner, of the Increasing, and Decreasing of the dayes and nights throughout the whole year.*

When the Sunne is in the *Æquinoctiall*, it riseth, and setteth at 6 a Clock, for in the instrument, the intersection of the *Æquinoctiall*, and the *Ecliptic* with the *Horizon*, is in the 6 a clock Circle on both sides. But if the Sunne bee out of the *Æquinoctiall*, declining toward the North, the intersections of the Parallel of the Sunne with the *Horizon*, is before 6 in the Morning, and after 6 in the Evening: and the diurnall Arch of the Sunne, greater then 12 howers; and so much more great, the greater the Northern Declination is. Again if the Sun be declining toward the South, the intersections of the Parallel of the Sunne with the *Horizon*, is after 6 in the Morning, and before 6 in the Evening; and the diurnall Arch lesser then 12 howers; and by so much lesser, the greater the Southern Declination is.

And in those places of the *Ecliptic* in which the Sun most speedily changeth his Declination, the length also of the day is most altered, and where the *Ecliptic* goeth most parallel to the *Æquinoctiall* changing the Declination but little, the length of the day also is but little altered.

As for example, when the Sunne is nere vnto the *Æquinoctiall*, on both sides, the dayes increase, and also decrease suddenly and apace: because in those places, the *Ecliptic* inclineth to the *Æquinoctiall* in a manner like a straight line, making sensible declination. Again when the Sunne is nere his greatest Declination, as in the height of the Summer, and the depth of Winter, the dayes

dayes except for a good time, as it were at one itay, because in those places the Ecliptic is in a manner parallel to the *Æquinoctiall*, scarce altering the declination: and because in those two times of the yeare, the Sunne standeth as it were still, at one declination; they are called the *Summer Solstice*, and the *Winter Solstice*. And in the meane places, the neerer every place is to the *Æquinoctiall*, the greater is the diversitie of dayes.

Wherefore we may hereby plainly see, that the common received opinion, that in every moneth, the dayes doe equally increase, is erroneous.

Also wee may see that in Parallels equally distant from the *Æquinoctiall*, the day on the one side, is equall to the night on the other side.

IX. Vse. *To find the Ascensionall difference of the Sunne every day.*

Seeke out the time of the Suns Rising, or Setting that same day (by the VII Vse) and see how much it differeth from sixe a clocke, then convert the same difference into degrees (as was taught in 1 Part. Chap. 12. Sect. 1.) by multiplying the howers with their decimall parts, by 15. And so have you the Ascensionall difference for that day.

X. Vse. *To find out the Oblique ascension of the Sunne every day.*

Seeke out the Sunnes Right ascension (by the IIII Vse) and the Ascensionall difference (by the IX Vse:) And if the Sunne be in the first Semicircle of the Ecliptic, Subtract the Ascensionall difference, out of the Right ascension: But if the Sunne be in the second Semicircle of the Ecliptic, adde the Ascensionall difference to the Right ascension: and you shall have the oblique ascension.

R

XI Vse

XI. Vse. *To find how farre the Sunne riseth, and setteth, from the true East and West points, which is called the Sunnes Amplitude ortive, and occasiue.*

Seeke out (as was shewed in the VI. Vse) the imaginary Circle, or Parallel of the Sunnes course, and the points of that Circle in the Horizon, on the East, and West sides, cutteth the degree of the Amplitude Ortive, and occasiue.

XII. Vse. *To find the length of every day and night.*

Double the hower of the Sunnes setting, and you shall haue the length of the day; and double the hower of the Sunnes rising, and you shall haue the length of the night.

XIII. Vse *To find the true place of the Sunne, vpon the Instrument, which answereth to the point, wherein the Sunne is in the heauens: and is the ground of all the questions following.*

Take with your Instrument the height of the Sunne, and reckon it on the moueable Index, or Labell: and then moue the said Labell, till you find the height of the Sunne, exactly to fall vpon the Parallel of the Sunne for that day, on the East side if it bee in the Fore-noone, and on the West side, if it bee in the After-noone; the point of intersection, where the Index, or Labell crosseth the Parallel

railel, in that point of the Sunnes altitude, shall bee the true place of the Sunne on the Instrument.

XIII. Vsc. *To find the Hower of the day.*

The true place of the Sunne on the Instrument (found out as was last shewed) sheweth among the hower lines the true hower of the day.

XV. Vsc. *To find out the Azumith or verticall Circle in which the Sunne is, or the Horizontall distance of the Sunne from the Meridian.*

The Index or Labell fastned at the Center, is a moueable Azumith: apply therefore the edge thereof, vnto the true place of the Sunne on the instrument (found out as was shewed by XIII. Vsc.) And make what point of the Horizon, or Limbe, the same edge of the Labell cutteth; reckon how many degrees of the Horizon, are intercepted betweene that point, and the Meridian line, or South point, either on the East, or West side: and that Arch shall be the Horizontall distance sought for, whereby is shewed the Azumith of the Sunne at that instant: and consequently the Angle which the verticall Circle, or Azumith of the Sunne maketh with the Meridian.

XVI. Vsc. *The Azumith of the Sunne being knowne, to find out the Altitude of the Sunne, and the Hower of the day.*

Set the edge of the Labell to the Azumith given, and make in what point the same edge crosseth the Parallel of the Sunne for that day: that point of intersection,

sheweth the height of the Sunne about the Horizon, vpon the Labell: and also it sheweth the hower of the day among the hower lines.

XVII. Vse. *To find at what hower the Sunne commeth to be full East, or West every day in Summer.*

Apply the edge of the Labell, vnto the East, or West points of the Limbe, and marke in what point, the said edge cutteth the Parallel of the Sunne for that day, for that same point among the hower lines, shall shew the time of the Sunnes coming to be full East, or West in that day, and likewise of what altitude the Sunne will be about the Horizon, at that time of his being full East, or West.

XVIII. Vse. *To find the height of the Sunne at high Noone every day, and likewise at every other hower.*

Marke in what point the Parallel of the Sunne for that day, cutteth the line of that hower, for which you would know the Sunnes altitude: And vnto that point of intersection, apply the edge of the moucable Labell, or Index: and thereon shall you find, the very degree of the Sunnes altitude, at that hower.

By this XVIII Vse, and by the XVI, are made the Quadrants, described by *Gemma Frisius*, *Munster*, *Clavius*, Mr. *Gunter*, and others: and also all manner of *Rings*, *Cylinders*, & innumerable other *topical Instruments*, for the finding out of the hower, and other like conclusions. And likewise the reason, of finding the hower of the day, by a mans shadow, or by the shadow of any Gnomon, set vp perpendicular to the Horizon, or else parallel to it.

XIX. Vse.

XIX. Vse. *To find out the Meridian line, and the points of the compasse without a Magneticall needle, yea more exactly then with a needle.*

Take the height of the Sunne, by the shadow of the pinne: and apply the same height, reckoned on the Index, or Labell to the parallel of the Sunne for that day, where by you haue the true place of the Sunne, in the instrument, as hath beene shewed in the XIII. Vse. Then keeping both the Labell at that point, and the pinne vpright in the Center, hold, or set your instrument parallel to the plaine of the Horizon, with the pinne toward the Sunne, and moue it gently, till the shadow of the pinne shall fall, exactly vpon, the fiduciall edge of the Labell. For then the Meridian line of the instrument, shall be in the true Meridian of the place: and the foure quarters of the instrument, shall looke into the foure cardinal points, of East, West, North, and South. Wherefore if with a bodkin, you make a prick at each end of the Meridian of your instrument where it standeth: and with a Ruler draw a line through them: the same shall bee the Meridian of that place.

This is a most excellent practise, for finding out the Meridian in any place, and is in an instant performed, and that easily. And hereby you may examine the *Variation of the Compasse*. And also exactly place any Sunne Dyall.

XX. Vse. *Considerations for the vse of the instrument in the night.*

In such questions as concerne the night, or the time before Sun rising, and after Sunne setting, the instrument representeth the lower Hemisphere, wherein the Southerne

Pole is elevated. And therefore the Parallels, which are about the *Æquinoctiall*, shall bee for the Southerne, or Winter Parallels, and those beneath the *Æquinoctiall*, for the Northerne, or Summer parallels. And the East shall be accounted for West, and the West for East: and the North shall bee accounted for South, and the South for North: contrary to that which was before, when the Instrument represented the vpper Hemisphere.

XXI. Vse. *To find* how many degrees the Sunne is vnder the Horizon, at any time of the night.

Seeke the declination of the Sunne for the day proposed: and at the same declination, on the contrary side, imagine a Parallel for the Sunne that night: and marke what point of it is in the very hower and minute proposed: then set the Index, or Labell to that point of the Parallel, and it will shew you thereon the degree of the Sunnes depression vnder the Horizon.

XXII. Vse. *To find out the length* of the Crepusculum, or Twilight every day.

Because the question concerneth the night time, you must seeke out the Sunnes Parallel, for the night, on the other side of the *Æquinoctiall*, hauing the same declination with that which the day of the moneth sheweth: then moue about the Labell, vntill the said Parallel cutteth the edge thereof in the 18th deg: on the West side for the Morning Twilight, and on the East side for the Evening Twilight, of the same day.

And note that in the height of Summer, the Twilight in our Horizon, continueth all night long: because
the

the same goeth not vnder the Horizon, full 18 degrees

XXIII. Vse. *To find the Declination of any Wall, or Plaine.*

Take a board hauing one streight edge, and a line drawne perpendicular vnto that edge: apply the streight edge vnto the Wall, at what time the Sunne shineth thereon, holding the board parallel to the plaine of the Horizon: and hang vp a thread with a plummet, so that the shadow of the thread may fall on the board, crossing that perpendicular line. Then take with your Instrument the height of the Sunne, and instantly make two prickes, in the shadow of the thread on the board, a good way distant one from the other: and laying a Ruler to those two prickes, draw a line, which line shall be the Azumith of the Sunne, on the board: againe with the height of the Sunne lastly taken, find out on your instrument, the Azumith of the Sunne; or the Angle which the Sunnes Azumith maketh with the Meridian, (by the XV. Vse.) And on the board taking the interfection of the shadow line with the perpendicular for the Center, describe a Circle equal to the innermost Circle of the Limbe: (which you may easily doe, if you set one foot of your compasses vpon the East, or West point, and extend the other foot vnto 60 degrees, on the same innermost Circle, for this distance is equal to the *Radius* thereof.) Againe with your compasses, take of the Arch betweene the Azumith of your Instrument, and the Meridian, and set that on the Circle of the board, that way that the true South is: and through the end of that Arch measured on the board, draw a streight line for the Meridian. Lastly take with your compasses, the Arch intercepted between the Meridian on the board, and the perpendicular line, and by applying it to the inmost Circle of the limbe, from the East, or West points see how many degrees it con-

containeth: for that is the declination of the Wall. Or else you may find the Meridian vpon the board, by X X Vse.

If the Angle of the Meridian with the perpendicular, on the board, be a right Angle, the Wall is direct East, or West.

But if the Meridian fall vpon the perpendicular, or be parallel thereto making no Angle with it, the Wall is direct North, or South.

XXIII. Vse. *The Art of Dyalling.*

And first how to make the Instrument in paper, promised in the beginning of this second part.

For the Delineation of this instrument in paper, it will bee necessary first to shew the manner how the Semidiameter is to bee graduated, or diuided into degrees: and how the Centers, and Semidiameters, of the severall kinds of Arches are to be found.

Vpon halfe a sheet of strong large Dutch paper, (the larger, the better) draw two straight lines, making a right Angle neere one of the corners, the one through the length, and the other through the breadth of the paper; which two lines I therefore call the longer, and the shorter perpendicular.

Vpon the right Angle point, being the Center, with a Semidiameter equal to that by which you intend to delineate your instrument, describe a quadrant of a Circle: and on the point where it meeteth with the shorter perpendicular, draw a long tangent line parallel to the longer perpendicular.

Divide the Quadrant into 90 degrees, among which from the beginning at the shorter perpendicular, reckon the elevation of the Pole, for which you will make your

instrument, and applying a Ruler to the end thereof, and to the Center, where the Ruler cutteth the tangent line make a prick. And taking with your compasses the distance from the Center to that prick, measure it vpon the shorter perpendicular: this shall be the *Semidiameter of the first lower Circle*. At the end thereof draw another long line parallel also to the longer perpendicular.

Then out of the Center vnto the second parallel through every degree of the quadrant, draw fine straight lines, cutting also the first Parallel. The intersection of those lines with the first Parallel, shall be *The scale of centers of Arches*. And their intersection with the second Parallel shall be *The scale of centers of lower Circles*. And the segments of those lines, intercepted betweene the Center, and the first Parallel, shall be the *Semidiameters of Arches*: and the whole lines betweene the Center and the second Parallel, shall bee *The Semidiameters of lower Circles*. And that you may know for what Circle, every Center, and Semidiameter serueth, you shall note every fift line from the beginning, with the figures 5, 10, 15, 20, &c. Set vnder the second Parallel, vnto 90 which will fall vpon the longer perpendicular: that so you may readily find the Center, and Semidiameter of any Circle required.

Againe divide the first 45 degrees of the Quadrant in the middest: and applying your Ruler to the Center, and to every one of those halfe divisions, where in each place the Ruler cutteth the first Parallel, or tangent line, make a prick. So shall you haue vpon the tangent line betweene the shorter perpendicular and the midlemost line 45, a third scale, which is, *The scale of 90 degrees*, for the graduating of the Semidiameter of your instrument on the paper: In which you shall also distinguish every fift degree, with figures set vnder the tangent line.

Having thus prepared your paper of scales with lines

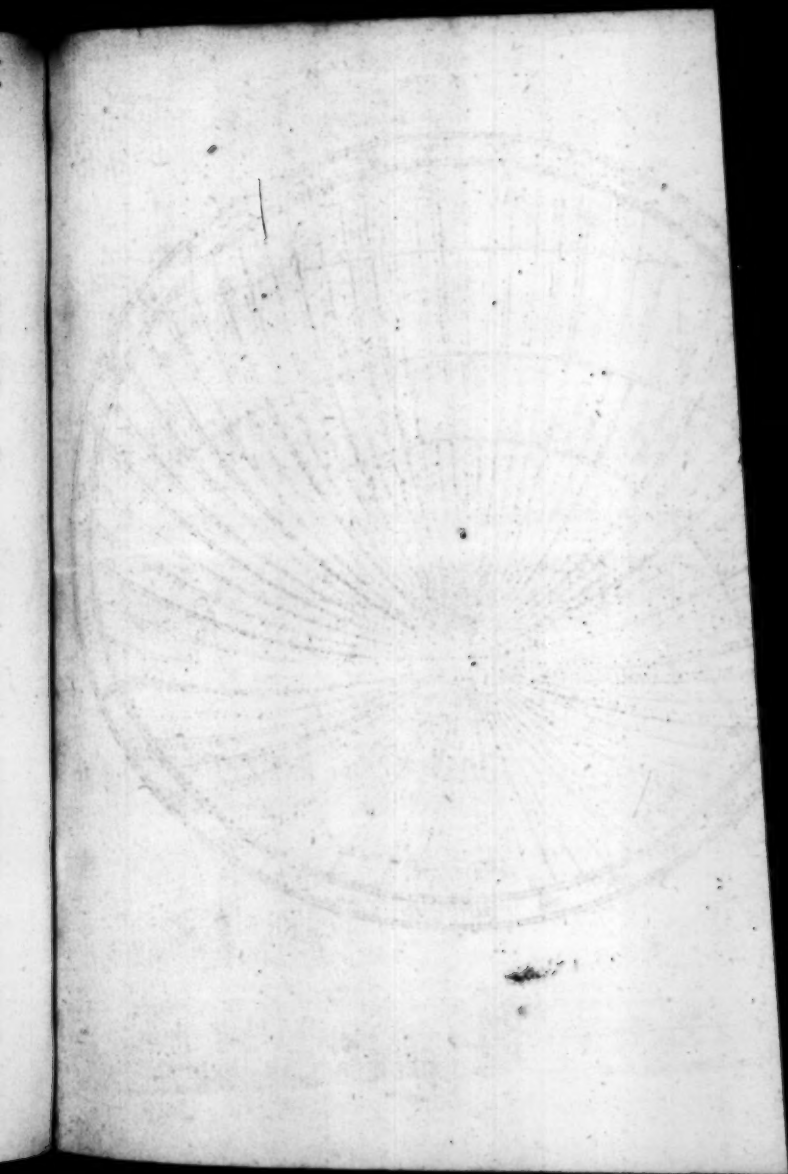
neatly and exactly drawne, keepe it by you to haue it still in a readinesse for the making, and using of the Instrument in paper. The making whereof is thus.

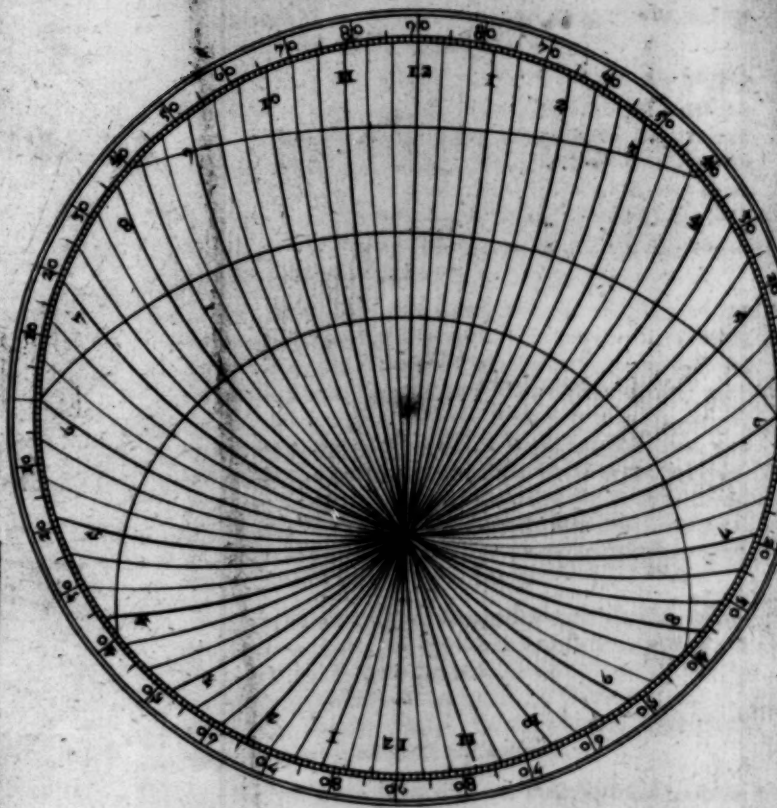
Take with your compasses the Semidiameter of the Quadrant in your paper of scales: and therewith vpon a peice of strong Dutch paper, Describe the Horizontall Circle: which you shall cut into two Semicircles with a Meridian line drawne through the Center: diuide them into Quadrants in the points of East, and West: and each Quadrant into 90 degrees to be marked with figures iust as is done in the Instrument.

Then with your compasses take the elevation of the pole vpon the scale of degrees in your paper: & set it vpon the Meridian line from the Center which way you please: that shall bee the intersection of the *Æquinoctiall* with the Meridian. Also reckon the complement of the height of the Pole, vpon the scale of Centers of Arches, and with your compasses take the distance from the end thereof to the Center: the same shall bee the Semidiameter of the *Æquinoctiall*, to bee drawne from the East point of the Horizon through the point of intersection with the Meridian vnto the West point.

Againe take with your compasses vpon the scale of degrees in your paper the complement of the height of the Pole: and set it vpon the Meridian on the other side of the Center from the *Æquinoctiall*: there shall bee the Pole of the *Æquinoctiall*, or of the World, in which all the houre lines shall crosse one another.

Nextly vnto the height of the Pole both adde, and also subduct $23\frac{1}{2}$ degrees: and with your compasses take both those Arches in the scale of degrees: and set them in the Meridian from the Center, one falling beyond the *Æquinoctiall*, and the other short of it: those shall be the intersections of the two Tropics with the Meridian. Seek also





and by Sect: 8, Chap: 12 of the first part, the Amplitude or rise of the Sun, having $33\frac{1}{2}$ degrees declination: and reckon it being found, in the Horizontall Circle from the East and West points both wayes: those are the points of intersection of the Tropics with the Horizon on both sides: and so having three points in each Tropic, you may easily through them drawe the Circles.

Moreover with your compasses, take the distance betweene the Center and the second Parallel in your paper, which is the Semidiameter of the sixt houre Circle: and set it on the Meridian from the Pole beyond the *Æquinoctiall*: that shall be the Center of the sixt houre Circle: vpon which you may draw the same Circle, from the East point of the Horizon through the Pole to the West point. Then through the center of the sixt houre Circle erect a line perpendicular to the Meridian, extending it infinitely on both sides of the Meridian: and in that line both wayes, pricke downe the Centers of the horary Circles, out of the scale in paper: And lastly opening your compasses from every one of those Centers vnto the Pole severally, describe all the horary Circles, or at least every fift of them, and so is your paper instrument perfectly finished.

The vse of this instrument on paper is, that lines, and arches may bee designed vpon it with a fine pennicell of blacke lead, and after ward be wiped out againe. Wherefore it will bee needfull for him that will vse this instrument, to all the purposes thereof, to get a good paire of large compasses with three points, one sharpe, another for lnke, a third for blacke Lead. And I suppose it would doe well to fasten over your instrument a peece of thinn coyled paper, through which the lineaments may be conspicuous: and vpon it to trace such lines, and arches as you haue occasion to vse: that so your instrument may be kept cleane, and last longer.

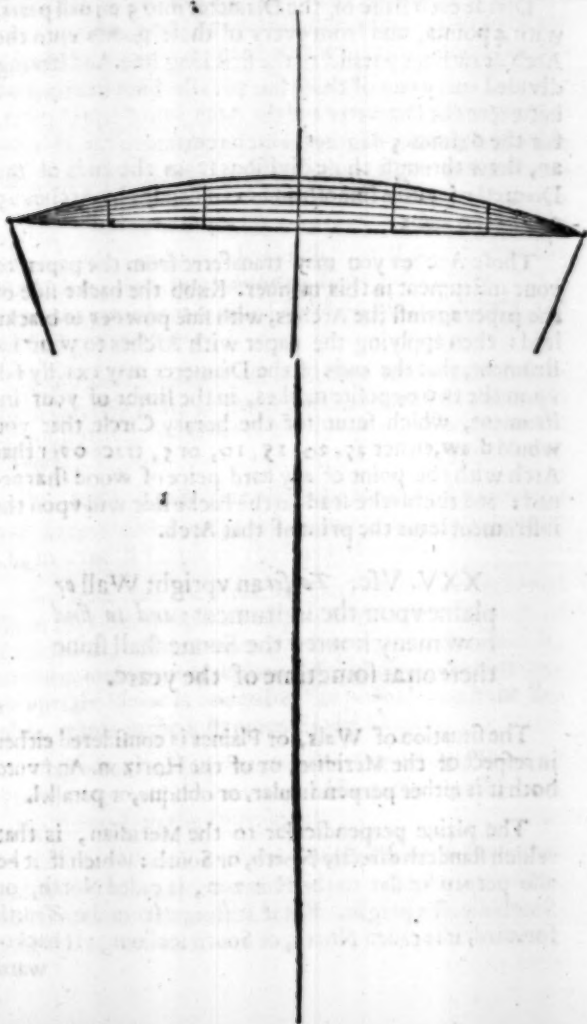
For as much as in delineating the horary Circles, which are within 30 degrees of the Meridian, the Semidiameters will be too long for your compasses: you may in that streight thus helpe your selfe. First say,

As the Radius,
is to the Sine of the Elevation of the Pole;
So is the tang: of the distance of any Horary
Circle from the Meridian, suppose 25,
or 20, or 15, or 10, or 5 degrees,
to the tang: of 30° to the Arch of the Horizon betwene the
Meridian, and that horary Circle.

Reckon this distance on the limbe of your Instrument from both ends of the Meridian, and marke it. Thus doe for the 25th, 20th, 15th, 10th, and 5th horary Circle on both sides of each end of the Meridian.

Then in any peice of cleane paper, through the midst of the longer way, draw a line: and toward one end which (I call the upper ende) crosse the same with a perpendicular line exactly equal to the Diameter of your Instrument, the point of Intersection being the center.

Take with your Compasses out of the paper of scales, the semidiameter of 60. degr: (which you may well doe for an ordinary instrument): and setting one foot on either end of the Diameter, that point wherein the other foot shall cutt the first long line, make your Center, and thereon draw an Arch through both ends of the Diameter, and cutting the vpper part of the first long line: this Arch is equal to that horary Circle, which is distant from the Meridian 30 degrees the complement of 60.



Divide each halfe of the Diameter into 3 equall parts, with 4 points, and from every of those points vnto the Arch draw lines parallel to the first long line. And having divided every one of those five parallel lines intercepted between the Diameter and the Arch into 6 equall parts, for the 6 times 5 degrees which remaine to the Meridian, draw through those divisions from the ends of the Diameter (with a smooth and even hand) the Arches 25, 20, 15, 10 and 5.

Those Arches you may transferre from the paper to your instrument in this manner. Rubb the backe side of the paper against the Arches, with fine powder of blacke lead: then applying the paper with Arches to your instrument, that the ends of the Diameter may exactly fall vpon the two opposite markes, in the limbe of your instrument, which serue for the horary Circle that you would draw, either 25, 20, 15, 10, or 5, trace over that Arch with the point of any hard peice of wood sharpened: and the blacke lead on the backe side will vpon the instrument leaue the print of that Arch.

XXV. Vse. *To set an vpright Wall or plaine vpon the instrument: and to find how many houres the Sunne shall shine thereon at some time of the yeare.*

The situation of Walls, or Plaines is considered either in respect of the Meridian, or of the Horizon. And vnto both it is either perpendicular, or oblique, or parallel.

The plaine perpendicular to the Meridian, is that which standeth directly North, or South: which if it be also perpendicular to the Horizon, is called North, or South direct vpright. But if it steepe from the Zenith forward, it is called North, or South inclining: If backeward

ward, it is called North or South reclining. And note that in a sloping Plaine that side which is toward the Horizon, is inclining, and that which is toward the Zenith is reclining.

The *Plaine oblique to the Meridian* is that which standeth not directly North, or South, but declineth one side into the East, and the other into the West: and is therefore called *Declining Eastward, or Westward*, according as either side of the Plaine looketh: As if an vpright Wall being Southerne, declineth from the South into the East, it is called *South declining Eastwards vpright*. But if it be not vpright, it is called *South declining Eastward, and inclining, or reclining*.

The *Plaine parallel to the Meridian*, is that which looketh directly East, or West; and accordingly, hath his denomination, whether it be *Vpright, Inclining, or Reclining*.

The *Plaine Parallel to the Horizon*, is called *Horizontall*; and is represented by the instrument it selfe, or at least by the innermost Circle of the limbe thereof.

And note that the Arch of Declination, is reckoned from the next East, or West point. And that the Arch of Inclination, or Reclination is reckoned from the Zenith, or the complement of it from the Horizon. So that every vpright Plaine is understood to passe through the Zenith, which in the instrument is the Center.

And thus having shewed the severall affections of Plaines, wee will now proceed to shew the manner how to set them vpon the Instrument.

A *Direct North, or South vpright Plaine*, is represented in the Instrument by a line drawne through the Center from the East point to the West, which is also the Horizontall interfection of the Plaine. And by it you shall

shall see that the Southerne side or face of the plaine is open to all the houres betweene fixe in the morning and fixe in the evening. And that about *London*, the Northerne side, onely in the Summer enioyeth the Sunne from his rising till after seven in the morning: and from before 5 a clocke in the afternoone, till his setting.

A direct East, or West upright plaine, is represented in the Instrument by the Meridian, which is also the Horizontall intersection of the plaine. And in it you shall see that all the forenoone houres are open to the East side: and all the afternoone houres to the West side.

A Declining Plaine is thus set upon the Instrument, reckon on the Horizon the arch of Declination, from the East, or West point: and at the end draw a line through the Center vnto the opposite point of the Horizon: So that each side thereof may be open to that point, either East, or West, into which the Declination is supposed. That line so drawne through the center is the Horizontall intersection of the plaine, and representeth the plaine it selfe, if it bee vpright. For example, there is about *London* an vpright Wall declining Eastwards 35 degrees: which I would set vpon the Instrument. Hold the Southerne part of the Instrument to you, and reckon from the East backward into the North vpon the Horizon 35 degrees: there draw a line through the Center: this line shall not onely vpon the South side represent a Southerne Plaine declining Eastward 35 degrees. But also vpon the North side shall represent a Northerne Plaine declining Westward 35 deg. And moreover it will appeare that on the Southerne side shall bee drawne the houres from almost 4 a clocke in the morning, till 3 in the afternoone. And that in the Northerne side shall bee drawne vpon one side 4 a clocke in the morning onely: and vpon the other side all the houres from 3 in the afternoone till Sunne set.

And

And so consequently the declination of an vpright wall, or Window being given, it may be found at what houre the Sunne vpon any day in the yeare will come to that Wall, or Window, and when it will goe from it. As in the former example, There is about *London* a Northerne wall declining West ward 35 deg. I would know at what time of the day the Sunne will begin to shine vpon it on the 24th day of *March*. Set the Index at 35 deg: from West toward South: and because that day the Sunnes Declination is 6 degrees Northward; Looke at what houre the sixt Parallel about the *Æquinoctiall* toward the Center meeteth with the Index so placed: and you shall find it at 3 a clock in the Afternoone. Wherefore at that time the Sunne will begin to shine vpon the Wall that same day.

The Poles of every vpright Wall are in the Horizon 90 deg: that is a quarter of a Circle, distant from the line representing the Plaine. Wherefore if vpon that line in the Center you erect a perpendicular, the ends thereof in the Horizon shall be the poles of that Plaine: and are so farre distant from the North and South points, as the Plaine it selfe is from the East, and West.

XXVI. Vse. *To set an Inclining, and Reclining Wall, or Plaine vpon the Instrument: and to find how many houres the Sunne shall shine thereon, at some time of the yeare.*

When you haue an *Inclining*, or *Reclining Plaine* to be described on the Instrument. First the Horizontall intersection is to be set thereon, as if it were vpright; together with the line perpendicular thereto, in which are the Poles of the Plaine: according as was taught in the XXV Vse.

Then vpon the scale of degrees in your paper, reckon the arch of Inclination, or Reclination; and with your compasses take, & set it in your Instrument vpon the line perpendicular to the Horizontall intersection of your Plaine, from the Center that way into which the Inclination, or Reclination tendeth: the same shall bee the vppermost point of your Plaine.

Againe, with your Compasses take the Complement of inclination, or reclination, both vpon the scale of degrees, and also vpon the scale of centers of arches in your paper: and set both spaces vpon the same perpendicular line, but on the other side of the center (extended if need be): At the shorter of those spaces shall be the pole of your plaine: and at the longer of them shall be the Center of it.

Lastly, setting one foot of your Compasses in the center of your Plaine, and extending the other foot to the vppermost point, describe in your Instrument an Arch of a Circle: which if you haue done well, will exactly fall vpon the ends of the Horizontal intersection of your Plaine. That Arch shall represent your Plaine, inclining vpon the lower side, which is toward the Horizon, or Limbe: but reclining vpon the vpper-side, which is toward the Zenith, or Center. And to either side shall shew in what hower lines the Sunne, at some time of the yeare, will shine vpon it: that in delineating a Dyall thereon, it may not be combered with vnecessary houre lines. For Example, suppose that the former Plaine, which with the South declined Eastward 35 deg: doe also incline 41 deg: 30 min. Wherefore also with the North side it shall decline Westward 35 deg: and Recline 41 deg: 30 min. Describe this plaine vpon your Instrument with an Arch of a Circle, found out as was taught last before. And it will appeare that vpon the Inclining side shall be drawne all the houre lines from almost 4 in the morning, to 4 in the afternoone: And vpon the Reclining side shall be drawne first 4 and 5 in the morning: and then beginning at 6 a clocke, all houres to Sunne set.

And

And note that in all Northerne plaines, the North Pole is elevated: and in all Southerne plaines the South Pole is elevated. Except such North inclining, and South reclining Plaines, that in the Instrument fall below the North Pole, betweene it and the Horizon: For in them the contrary Pole is elevated. And also that a direct East and West plaine, if it Recline, hath the North Pole elevated: and if it Incline, the South Pole.

XXVII. Vse. *The Plaine being set vpon the Instrument, to find the distances of the houre lines, and Substile from the Horizonall Interfection. And also the height of the Stile aboue the Substile.*

Euery Dyall either hath a Center in which all the houre lines, together with the Substile, and Stile doe meet: or else it hath no center, & so they are al parallel one to another.

If the Plaine being set vpon the Instrument, cutteth the Pole of the *Æquinoctiall* (that is the point in which all the horary Circles crosse the Meridian) the Dial to be drawne vpon that Plaine shall haue no Center. But if it cutteth not the Pole, the Dial shall haue a Center. And of these Dyals with Centers wee will first in:reate: as being most proper for the vse of the Instrument.

Behold therefore the Pole of your plaine heedily what horary Circle it falleth vpon: Or if it fall betweene any two, the distance of each being reasonably apporportioned, imagine a horary Circle passing through it. Marke in what point that horary Circle, either real, or imagined, doth cut the Plaine, that same point shall bee the place of the Substile in your Plaine: and the height of the Stile aboue it, is the Arch of that horary Circle intercepted betwixt the Pole of the *Æquinoctiall*, and the point of the Substile noted in the Plaine.

Therefore applying a Ruler to the Pole of your Plaine, carry it about vnto all the interfections of the Plaine with the houre Circles, and the substile leuerally: and where in euery place the Ruler shall cut the innermost Circle of the Limbe, there make a visible marke: For the arches of the Limbe intercepted betweene the Horizontall points of your Plaine, and euery one of those marks, shall be the distance intended to be sought.

But for the Horizontall plaine, the ends of the houre Circles in the Limbe of your instrument, doe giue the distance without any more adoe.

Concerning the height of the Stile above the Substile: It is apparent by the instrument, that in a Horizontal dial, the 6 a clocke line lyeth directly East and West: and the Meridian perpendicular to it, directly North and South. And that the Meridian is the Substile. And that the height of the Stile above the Substile, is equal to the height of the Pole in that place.

It is also apparent, that in all direct North and South Dyals, the 6 a clocke Line is drawne parallel to the Horizon, and the Meridian perpendicular to it: And that the Meridian is the substile. And that the height of the Stile above the substile, if the Plaine bee vpright, is equal to the complement of the height of the Pole. But if it be North inclining, or South Reclining, it is equal to the difference, of the height of the Pole, and the Arch of Inclination, or Reclination. And if the Plaine bee North Reclining, or South Inclining, it is equal to the Summe of the complement of the height of the Pole and of the Arch of Inclination, or Reclination. And if the Plaine fall vpon the Equinoctiall, the stile shall stand vp perpendicular vpon it in the Center: and the hower lines shall be drawne all at equall Angles, *viz.* 15 degrees one from another.

In such Dyals as haue not the Meridian for the stile, the height of the stile aboue the Substile is thus found by the Instrument. It was shewed before that the height of the stile aboue the substile is the Arch of the horary Circle through the Pole of the Plaine, intercepted betweene the pole of the *Æquinoctiall*, and the point noted in the Plaine for the substile. Therefore from the Horizontall points, or interfections of that horary Circle reckon 90 degrees both wayes: and thereto through the Center draw an obscure line: in which line shall be both the Inclination of that horary Circle, which is the distance of the interfection, or vppermost point thereof from the Center: and also the Pole. Then with your compasses take that distance, or Inclination: and setting it vpon the scale of degrees in your paper, see how many degrees it containeth vpon that scale. Againe vpon the same scale take with your compasses the complement of that inclination or distance, which being set vpon the obscure line on the other side of the Center, shall shew the Pole of that horary Circle. Lastly applying a Ruler to the Pole of that horary Circle, and both to the Pole of the *Æquinoctiall*, and to the point of the substile in the Plaine severally: marke where in both places the Ruler cutteth the innermost limbe of your instrument: For the degrees of the limbe intercepted betwixt those markes, shall be the height of the stile, aboue the substile, which was sought for.

And by this which hath beene taught, you shall find that in an vpright Dyall declining, as before 35 deg: from the South into the East, or from the North into the West, the substile shall fall vpon that horary Circle, which is about 3 deg: after 9 a clock in the morning: and the stile eleuated aboue the substile about 31 deg: And also that in a South Dyall declining Eastward 35 deg: and inclining 41 deg: 30 min: Or in a North declining Westward 35 deg: and reclining 41 deg: 30 min: the

substile shall fall vpon that horary Circle which is about 8 degrees after 6 a clock: and the stile eleuated about the substile about 63 deg. 30 minutes.

XXVIII. Vse. *The making of all manner of plaine Dyals with Centers.*

I haue already shewed how to find out in our Instrument the distances of all the houre lines, and the substile, from the Horizontal interfection. Now the delineating of a Dial is nothing else but to transerre those distances out of your Instrument into the Dial plaine, euery one in his due situation: and then through them, out of the Center, to draw such houre lines as shall be of vse, together with the substile.

The due situation of those distances vpon the Dial plaine, dependeth on the true placing of the Meridian, or 12 a clocke line: for that being truly described, all the rest will be easie enough.

First therefore I will shew the manner how the Meridian, or 12 a clocke line, is to be described.

Take in your Dial some point for the Center, where you shall thinke fit: and through it draw a line parallel to the plaine of the Horizon. Crosse it in the Center with a perpendicular line. And hauing opened your compasses to the length of the Semidiameter of your paper Instrument, describe on the Center a Circle equal to the innermost Limbe thereof. In which Circle the line parallel to the Horizon is for the Horizontal interfection: and the other for the line perpendicular to it: and the Circle it selfe representeth the plaine: Marke therein the East and West sides of the Plaine with *E* and *W*.

In the Horizontal, and in al North and South direct Plaines, both vpriight, and slooping; and in all vpriought

declining plaines, the Meridian is perpendicular to the Line parallel to the Horizon.

In North ~~inclining~~, and South ~~inclining~~ plaines, the Meridian is to be drawne on that side of the Dial plaine either East, or West, into which the declination is: But in North inclining and South reclining, on the contrary side. And if the plaine bee Northerne, the Meridian shall be above the Line parallel to the Horizon: and if the plaine be Southerne, it shall be vnder it. And if the contrary Pole be eleuated, it shall be drawne through the Center into the opposite Quadrant of the Circle in your Dial plaine.

Lastly in a direct East and West plaine, both inclining and reclining, the Meridian is the same with the line parallel to the Horizon.

Wherefore with your compasses take the distance in the limbe of your Instrument, from the next Horizontall point, vnto the marke of the Meridian; and measure it vpon the Circle of the Dial plaine, in that part, and on that side, according as in consideration of the eleuated Pole, and of the qualitie of the Plaine, was shewed to be agreeable. And at the end of that arch, through the Center, draw a line for the Meridian.

Again with your compasses take the distances in the limbe of your Instrument, betweene the marke of the Meridian, and the markes of all the houre Lines severally: and setting them vpon the Circle of the Dial plaine orderly from the Meridian, the Forenoone houres on the West side of it, and the Afternoone houres on the East side: at the end of euery one of those arches draw the houre Lines: and distinguish them with their proper figures accordingly.

Lastly fasten the stile in the Center, so that it may hang perpendicular vnto the plaine in the Substile, at the iust height. And because the stile in every Dial is vnderstood to be a segment of the Axis of the world
which

which is a line imagined to passe from the North to the South Pole through the Center of the earth; the stile being rightly placed shal still with the end point towards the elevated Pole, that is vpwward from the Center, if the North Pole be elevated; or downeward from the Center if the South Pole be elevated.

XXIX. Vse. *The making of all manner of plaine Dyals not hauing Centers.*

If the plaine represented on the Instrument (as was taught before in the XXV and XXVI Vses) cut the Pole of the Equinoctiall, it is an horary Circle, either one of them which are drawne in the Instrument, or falling betwene some two of them: and the Dyall plaine it selfe shall not crosse the axis of the world, but lye parallel to it, without any Angle of eleuation. And therefore such a Dyal can haue no Center: But the stile, the substile, and all the houre lines shall be parallel one to another.

Every such Plaine represented on the Instrument,

Either, *First* it is the Meridian of the place, the Horizontall intersection whereof is the 12 a clocke Line drawne from North to South: and the Dyall made thereon, is a direct East, or West vpright Dyal: In which the substile is distant from the Line, in the Circle of the Dyall plaine parallel to the Horizon, with an Arch equall to the eleuation of the Pole, and vpwward toward the Pole. And is also the 6 a clocke line in your Dyal.

The rest of the houre lines are thus described. Draw through the substile, in any point, a long Line at right Angles: that line shall bee the Equinoctiall intersection vsually called the *Contingent line*: And taking a conuenient distance for the stile to hang parallel over the substile (according to the greatnesse of your Dyall plaine)

measure

measure it vpon the substile from the *Equinoctiall* intersection: and vpon the end of that measure, describe halfe a Circle for the *Equinoctiall* it selfe. Diuide each Quadrant thereof from the substile, into 6 equall parts, or houres. Then applying a Ruler to the Center, and to euery one of those diuisions seuerally, where in euery place the Ruler shall cut the long line of *Equinoctiall* intersection, make pricks: and through those pricks draw the houre lines, parallel to the Substile, or 6 a clocke line: distinguishing so many of them as bee needfull, with their figures: that is all the Forenoone houres on the East plaine, and all the Afternoone houres on the West plaine. But in these Dyals there is no 12 a clock line, it being infinitely distant from the Substile. Lastly hang the stile directly over the Substile, and parallel to it, at the distance formerly taken. And thus are your East, and West Dyals finished.

Or *Secondly*, it is the first houre Circle, the Horizontal intersection whereof is the line of East, and West: and the Dial made thereon is direct North inclining, or South reclining, with an Arch equal to the complement of the height of the Pole. And the parallel to the Horizon is the *Equinoctiall* intersection: and the line perpendicular to it is the 12 a clocke line, and also the Substile.

The rest of the houre Lines, from 7 a clocke in the morning, to 5 in the euening, are thus described. Take a conuenient distance for the Stile from the Substile, measuring it vpon the Substile from the *Equinoctiall* intersection: and on the end of that space describe the Semicircle of the *Equinoctiall*, to bee diuided on both sides of the Substile into 6 houres: through euery one of which out of the Center, a Ruler being applyed; at the points of the seuerall intersections of the Ruler with the *Equinoctiall* intersection, draw the houre Lines parallel to the Substile, or 12 a clocke Line: distinguishing them with their figures, namely 11, 10, 9, 8, 7, on the West side:

and 1, 2, 3, 4, 5, on the East side: but in these Dyals there is no fine a clocke Line, it being infinitely distant from the Substile. Lastly hang the Stile directly over the Substile, and parallel to it, at the distance formerly taken.

Or *Thirdly*, it is North inclining, or South reclining, and also declining: in which.

As the tangent of the *Elevation of the Pole*, is to the *Radius*;

So is the Sine of the compl: of *Declination*, to the tang: of the compl: of *Inclination or Reclination*.

The Plaine being set vpon the Instrument by the Arches of Declination and Roofing thereof (as hath beene taught in XXV. Vie) shall cut the pole of the *Equinoctiall*. Apply therefore a Ruler to the Pole of the plaine, and to the Pole of the *Equinoctial*; and the point in which it cutteth the Limbe, marke for the substile: which is to bee transferred vnto the Circle of the Dyal plaine, by taking the distance betweene that point, and the next Horizontal intersection, and setting it on that Circle from the line parallel to the Horizon, vpward if the plaine be North: or downie-ward if the plaine be South: and on that side which is contrary to the Declination. The substile being thus found, draw a long line perpendicular to it, for the *Equinoctial* intersection. And taking a conuenient distance for the stile from the substile, measure it vpon the substile from the *Equinoctial* intersection: and on the end of that space describe the Semicircle of the *Equinoctial*. Then looke in your Instrument how many degrees of the *Equinoctial* are intercepted betweene the Meridian, and the Arch representing your Plaine: and reckoning the same number of degrees vpon the *Equinoctial* of the Dyal plaine, from the substile towards the side of Declination, there make a marke for the Meridian point thereof: in which you

you must begin to diuide the *Æquinoctial* semicircle into houres both wayes: And that being diuided, apply a ruler to the center, and to euery one of the diuisions: and at the points of the feuerall interfections of the ruler with the *Æquinoctial* interfection, draw the houre lines parallel to the substile. Set 12 at that houre line which was drawne at the interfection through the Meridian point of the *Æquinoctial*: and 11, 10, 9, 8, &c. on the West side: and 1, 2, 3, 4, &c. on the East side. Lastly, hang the stile directly ouer the substile, and parallel to it, at the distance formerly taken.

XXX Vsc. How by Sines and tangents to calculate the places of the Meridian, and Substile, and the height of the Stile above it: and the distance of the Meridian of the Æquinoctiall from the Substile; together with the places of houre lines, both by calculation, and also Geometrically.

I haue already taught the making of all manner of plaine Dyals most easily by the Instrument, for the same height of the Pole. But if any man either want an Instrument, or else desireth greater exactnesse, I will also here shew how to performe the same by calculation, on the other side of the Instrument.

In a plaine erect Dyall declining.

As the Radius

is to the Sine of declination;

So is the tang. of the compl. of the Poles height.

to the tang. of the distance of the

substile from the Meridian.

Againc

As the *Radius*

is to the Sine of the compl. of *Declination*;
So is the Sine of the compl. of the *Poles height*,
to the Sine of the *height of the stile*
above the substile.

Thirdly

As the Sine of the *Poles height*
is to the *Radius*;
So is the tang. of *Declination*,
to the tang. of the *Distance of the Meridian of the*
Equinoctial from the substile. And this distance
is ever lesse then 90 degrees.

In a plaine East and West Inclining
and Reclining Dyall.

As the *Radius*

is to the Sine of the compl. of $\left\{ \begin{array}{l} \text{inclination} \\ \text{reclination} \end{array} \right.$
So is the tang. of the *height of the Pole*,
to the tang. of the *distance of the substile*
from the Meridian.

Againc

As the *Radius*

is to the Sine of $\left\{ \begin{array}{l} \text{inclination} \\ \text{reclination} \end{array} \right.$
So is the Sine of the *height of the Pole*,
to the Sine of the *height of the stile*
above the substile.

Thirdly,

As the Sine of the compl. of the *Poles height*,
is to the *Radius*;
So is the tang. of the compl. of $\left\{ \begin{array}{l} \text{inclination} \\ \text{reclination} \end{array} \right.$
to the tang. of the *distance of the Meridian of the*
Equinoctial from the substile. And this distance is
ever greater then 90 degrees.

In plaine Dyalls both declining and also inclining, and reclining.

As the *Radiu*

is to the Sine of $\left\{ \begin{array}{l} \text{inclination} \\ \text{reclination} \end{array} \right.$

So is the tang: of *Declination*,

to the tang. of the compl. of the distance of the Meridian, from the line parallel to the horizon.

Againe

As the *Radiu*

is to the Sine of the compl. of *Declination*;

So is the tang. of the compl. of the Poles height,

to the tang. of *Base I*.

If the Dyall be South reclining, or North inclining, the summe of *Base I*, and of the complement of Inclination or reclination shall be *Base II*. But if the Plaine be South inclining, or North reclining, the difference of *Base I*, and of the complement of inclination, or reclination shall be *Base II*.

Then say thirdly,

As the Sine of the compl. of *Base I*,

is to the Sine of the compl. or excess of *Base II*;

So is the Sine of the height of the Pole,

to the Sine of the height of the stile
above the substile.

Fourthly,

As the Sine of the compl. of the height of
the stile above the substile,

is to the Sine of *Declination*;

So is the Sine of the compl. of the height of the Pole,

to the Sine of the compl. of the distance of the substile
from the line parallel to the horizon.

Fiftly

As the Sine of the compl. of the *bright of the stile*
above the *substile*,

is to the Sine of *Declination*;

So is the Sine of the compl. of $\left\{ \begin{array}{l} \text{inclination} \\ \text{reclination} \end{array} \right.$

to the Sine of the *distance of the Meridian of the*
Æquinoctiall from the substile.

And note that in South reclining, and North inclining
Plaines, if *Base II* be lesse then a quadrant, the contrary
pole is eleuated above the Plaine: And if *Base II* be equall
to a quadrant, the Plaine doth cut the Pole of the *Æqui-*
noctiall.

Now concerning the placing of the substile vpon the
Dyall plaine (as I haue already in the *XXVIII Vse* shewed
for the Meridian) Wee are to know, First that the sub-
stile is to be drawne vward from the line parallel to the
horizon, if the Plaine be Northerne; or downeward from
it, if it bee Southerne. Except in North reclining, and
South inclining Dyalls, in which the *Base I* exceedeth the
complement of inclination, and reclination: for in them
it is quite contrary. And secondly that the substile is to
be drawne in the contrary side from the Declination. But
in North inclining, and South reclining Dyalls, in which
the contrary Pole is eleuated, the substile must be drawne
through the center into the opposite quadrant of your
Dyall circle.

Lastly, the *houre lines* in all manner of plaine
Dyalls, are thus to be found.

If the substile and houre bee both on the same side of
the Meridian: the arch of the *Æquinoctiall* betweene the
substile, and the houre line, shall bee equall to the diffe-
rence of the two distances, namely of the houre line from
noone, and of the Meridian of the *Æquinoctiall* from the
substile. But if the substile bee vpon one side of the Me-
ridian,

ridian, and the houre on the other side : it shall be equal to the summe thereof. Then say

As the *Radius*

is to the *Sine of the height of the stile
above the substile,*

So is the tang. of the arch of the *equinoctiall, betweene
the substile and the houre line,*

to the tang. of the arch of the circle of your *Dyall
plane, betweene the substile and that lower line.*

Or else you may without calculation Geometrically inscribe the houre lines in Dyals hauing centers (for how to doe it in Dyals not hauing centers, I haue already shewed in the XIX *Fe*) thus.

Describe in your Dyall plane a line for the stile, at the same height or distance from the substile, that the true stile ought to haue. Take also in the substile (as in reason you shall see fit) a point, and through it draw at right angles a long line, for the contingent, or *Equinoctiall* intersection. Again from the same point let fall a perpendicular vnto the stile : the length of this perpendicular is the nearest distance betweene that point and the stile : and it is also the distance of the center of the *Equinoctiall* from that point : measure it therefore vpon the substile, the contrary way from the center of the Dyall : and hauing thus the center of the *Equinoctiall*, describe there vpon toward the contingent line one halfe of the *Equinoctiall* circle : which if the substile be the Meridian, or 12 a clock line of your Dyall, you must begin to diuide into houres at the substile : But if the substile and Meridian of your Dyall be severall lines, apply a ruler to the center of the *Equinoctiall*, and to the intersection of the 12 a clock line with the contingent, and there draw a line : this line shall bee the Meridian of the *Equinoctiall* : at which you must begin to diuide the *Equinoctiall* circle into houres, both wayes. Then applying a ruler vnto the center of the *Equinoctiall* & every one of those diuisions, where the

the rules in every place shal cut the contingent line, there make a mark: and lastly, through every one of those marks from the center of the Dyall, draw the houre lines themselves.

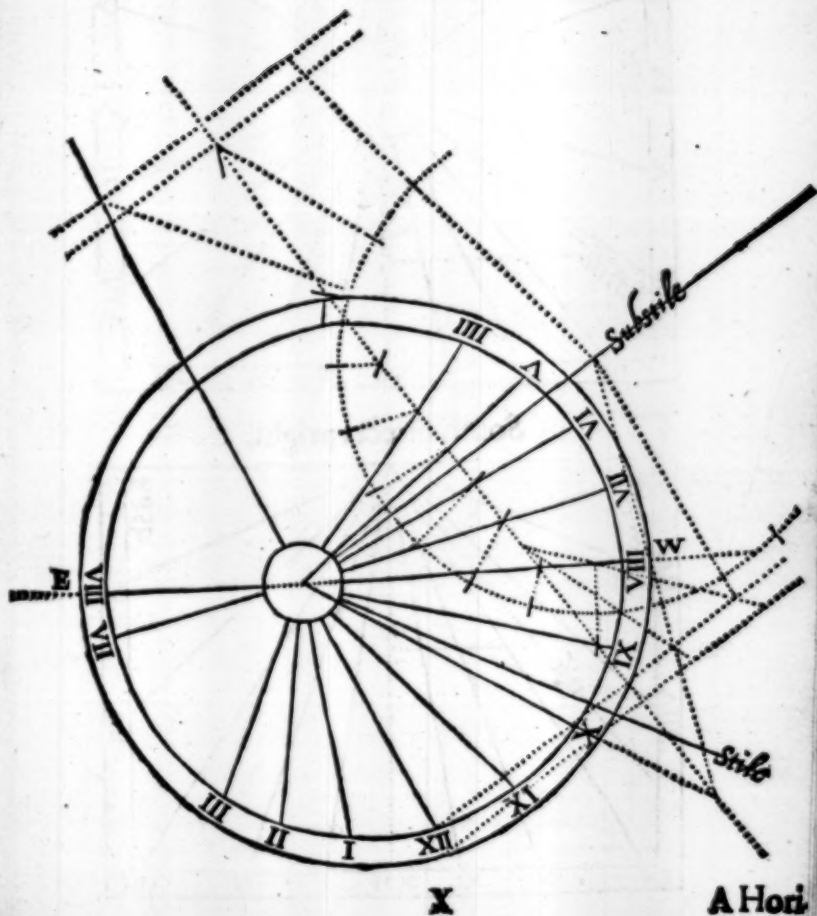
And if in any houre line it shall happen, that the ruler so applied will not reach to intersecat the contingent line: you may thus help your selfe. Which rule also may serue you to find the Meridian of the *Æquinoctial*, as often as the intersection of the Meridian of the Dyall with the contingent, falleth without your paper or plaine.

Draw the houre line as farre as it will goe. And take with your Compasses the distance of the intersection point of the contingent with the subtile, both from the center of the Dyal, & frō the center of the *Æquinoctial*. And taking at all adventure a point in the contingent line, on that side in which the houre line is, measure from that point on the contingent, both those distances: and at the ends of them both draw two lines parallel to the subtile, crossing the contingent. Then applying a ruler to the point, which you tooke at all adventure, and to the intersection of the parallel, which hath the distance of that center, whence the houre line giuen proceedeth, with that houre line: where the ruler shall cut the other parallel, make a prick: and measure the distance betweene that prick and the contingent, vpon the former parallel, on the other side of the contingent. Lastly, out of the proper center through the end of that measure, draw a line: which shall be that you desire.

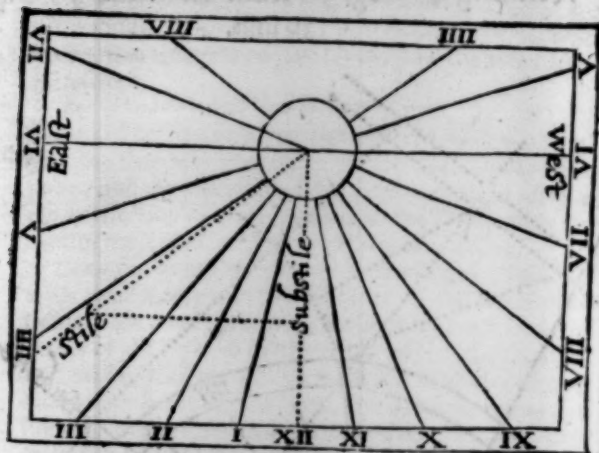
An example of this Geometrical way of delineating the houre lines you shall finde in the description of a South-^{Upright} North Dyall declining 35 degr. and reclining degr. 41 min. 30. by considering whereof these rules will be found exceeding plainly set downe: As also all the other rules and observations here delineared, so one that is any whit pregnant and ingenious, will neede no other exemplification, then the inspection of the instrument is selfe, and of these severall Dyalls following

FINIS.

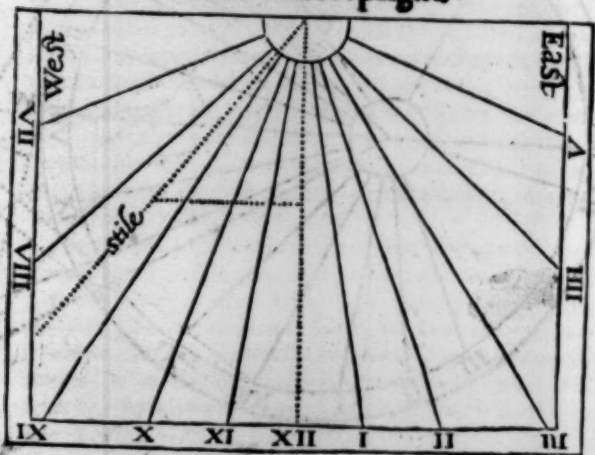
A North Dyall declining Eastwards 35 deg.
 reclining 41 deg. 30 min. Latitude 51 deg.
 30 min.



A Horizontall Dyall

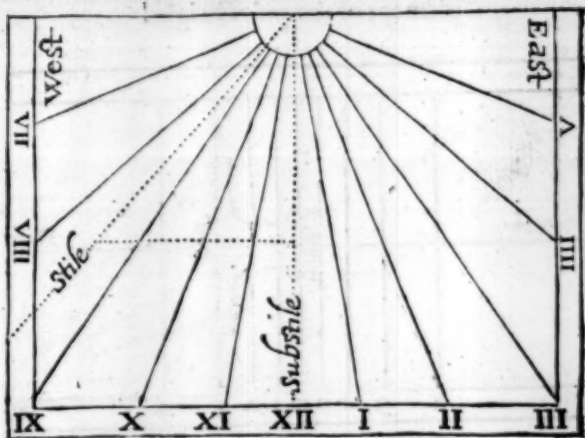


South direct vpright

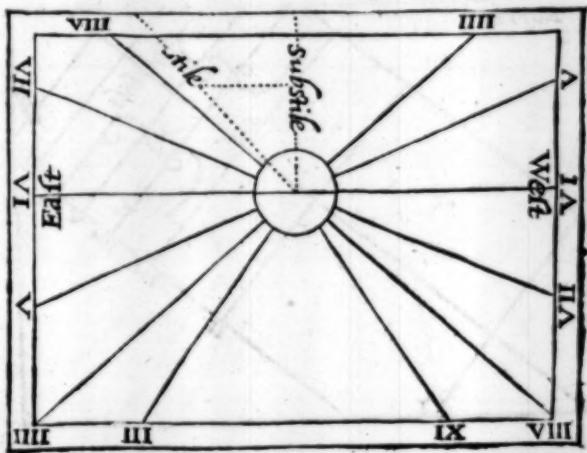


South

South direct inclining 24 deg.



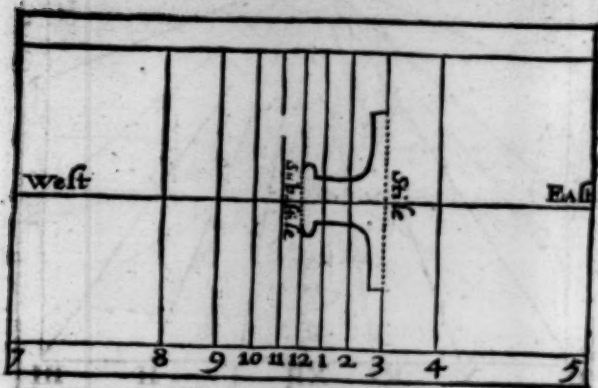
North direct reclining 24 deg.



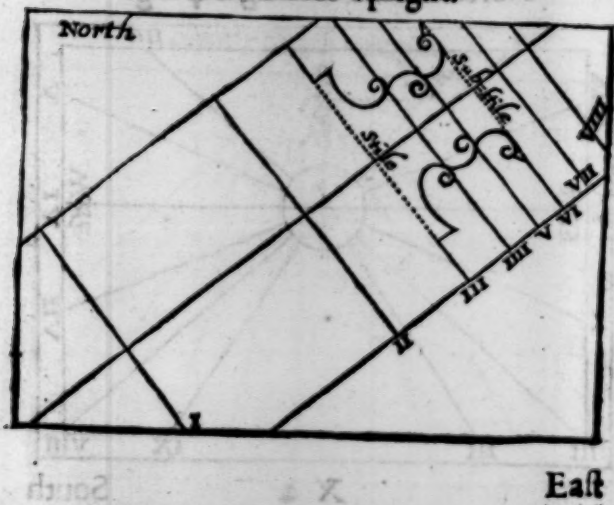
X 2

South

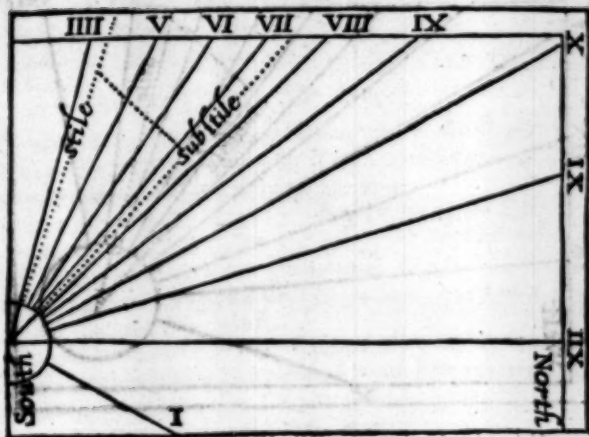
South direct reclining equall to the com-
plement of the poles height.



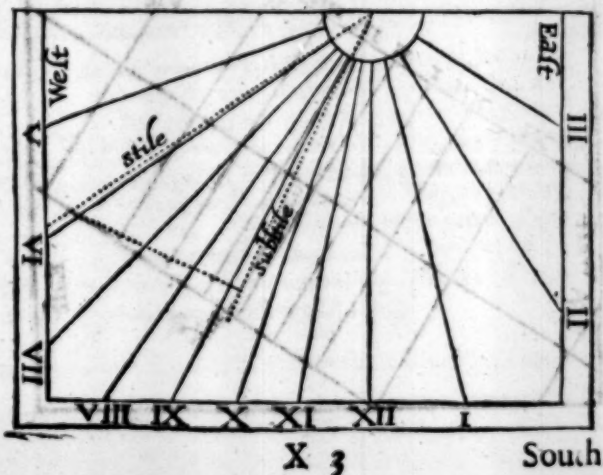
West direct upright.



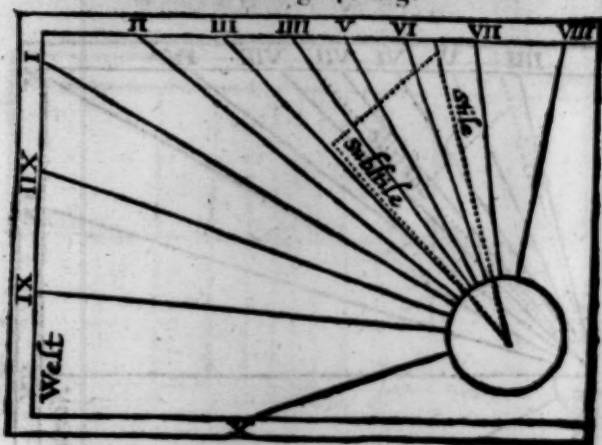
East direct reclining 3 2 deg.



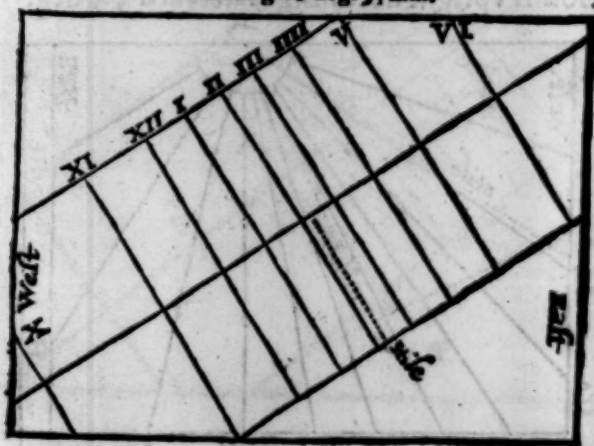
South vpright declining Eastward 35 deg.



South declining Westward 76 degr.
reclining. 48 deg.



South declining Westward 61 degr.
Reclining 21 deg. 31 min.



The Translator to the Reader.

Gentle Reader, by reason of my absence, whilst this Book was in the Presse, it is no meruaile though some faults haue escaped, which you will be pleased to amend them.

Pag. 14. lin. 14, 2.0413927. Pag. 15. lin. 1, the first terme of a progression Pag. 17, lin. 17, the Antecedent arme Pag. 18. lin. 19, terme giuen from Pag. 19 lin. 11, in the fift circle Pag. 20. lin. 19, lye hid. As in this lin. 20, D. Rat. mul^m — 1 in R.

lin. 28, and Rat mul^m in R in a — R in a,

Also in the aquations pag. 21, 24, 26 which haue a magnitude equal to a fraction: the same magnitude together with the note of equality, ought to be set right against the line that is betwene the Numerator and Denominator of the fraction, as in these, $\frac{\text{Rat. mul}^m - 1 \text{ in R in } a}{D} = Z$. And

$\frac{Z \text{ in } D}{a} = \text{Rat. mul}^m \text{ in R} - R$. And so of the rest.

Pag. 25 lin. 20, arme as 71, 381: Pag. 26 lin. 16, Ratiocination. Pag. 29 lin. 29, number of figures Pag. 35 lin. 5, 61, 174. Pag. 36 lin. 11, 1748 lin. 14, 1748. 3126 +. Pag. 44 lin. 8, is a 100fc line 26, therof Pag. 94 lin. 21, a circle, or 90 degrees. Pag. 100 lin. 3, and then the side DC Pag. 113 lin. 7, delineation Pag. 127 lin. 1, the Sunne goeth not Pag. 131 lin. 19, in the paper Pag. 132 lin. 10, to the tangent of the arch of Pag. 143 lin. 3, in North reclining, and South inclining Pag. 152 lin. 30 North Dyall declining Eastward 35 deg.